Design of a Disturbance Observer for a Two-Link Manipulator With Flexible Joints

Jong Nam Yun and Jian-Bo Su, *Senior Member, IEEE*

***Abstract*— This brief proposes a method for designing a disturbance observer (DOB) to decouple joint interactions in robot dynamics with nonlinearity. The traditional DOB based on filter design theory has limited performance since the cut-off frequency of its Q-filter is the only tunable parameter to deal with disturbance suppression and model uncertainty. In this brief, a robust optimal design method is developed for the DOB, which can achieve optimal performance of suppressing disturbance by systematically shaping its Q-filter. Simulation results of application to a two-link manipulator with flexible joints show the improvements in disturbance suppression, which illustrates the validity of the proposed method.**

***Index Terms*— Disturbance observer (DOB), H_∞ standard control problem, low-pass filter, robust motion control, two degrees-of-freedom.**

I. Introduction

IT IS ESSENTIAL to improve the robustness of a motion control system to external disturbance and parameter variations. Disturbance observer (DOB)-based control is one of the most popular methods to attain this purpose [1]. Recent results in experiments and applications have shown the effectiveness of DOB based control [2]–[6]. Especially, it is widely used to decouple the interactive model of robot manipulator, compensate its nonlinearity, and improve the speed and accuracy of control [7]. The DOB for this purpose is often designed by traditional filter models such as binomial model and Butterworth model [8]–[10], but these model have cutoff frequency as the only tunable parameter. In these models, To improve the performance of suppressing disturbance, the cutoff frequency should be increased. However, because of the existence of high frequency model uncertainty caused by flexibility of joint shafts and change of load etc., the cutoff frequency is restricted to guarantee robust stability. This is a substantial disadvantage of using conventional filter model.

Recently, several design methods of DOB using H_∞ control scheme have been reported, which can provide optimal performances of rejecting disturbance and sensor noise on the condition of guaranteeing robust stability to parameter uncertainty. However, most of the methods employ numerical computation algorithms to obtain optimal Q-filter [11]–[13]. A systematic and straightforward method is proposed in [14], where H_∞ optimization problem with structural restrictions of whole order, relative order and internal model order specifications is solved in the framework of standard H_∞ control problem. However, it only guarantees the robust stability of DOB local loop to parameter variations, but not that of closed-loop system.

In this brief, instead of conventional filter model, a method of designing robust optimal DOB is proposed, where Q-filter is designed by systematic and straightforward H_∞ norm optimization procedure. In this method, the performance of optimal disturbance suppression is achieved by directly shaping Q-filter itself, without tuning cutoff frequency. Thus, the disadvantage of conventional model could be overcome, resulting in attaining optimal disturbance suppression performance under the condition of satisfying robust stability of closed system against model uncertainty.

The remainder of this brief is organized as follows. Section II discusses some properties of frequency responses of sensitivity function and complementary sensitivity function in DOB system as a mixed sensitivity problem. Section III propose design method of robust optimal Q-filters of DOB that provides optimal disturbance suppression performance under the condition of satisfying robust stability against model perturbation caused by variations of parameter and load. Simulation results on application to the robust control of two-link manipulator are shown in Section IV followed by conclusions in Section V.

II. Analysis of Frequency Responses of DOB

The DOB design problem essentially can be regarded as weighted mixed sensitivity problem related to Q-filter [8]. In this problem, disturbance suppression performance depends on sensitivity function $S_{\mathrm{DOB}}(s) = 1 - Q(s)$ while noise rejection performance and robust stability to model uncertainty depend on complementary sensitivity function $T_{\mathrm{DOB}}(s) = Q(s)$ [9]. Therefore, in order to obtain optimal control performances, both $S_{\mathrm{DOB}}(s)$ and $T_{\mathrm{DOB}}(s)$ should be as small as possible. But, since sum of them is constant, it is impossible to minimize them over all frequencies. Therefore, weighted mixed sensitivity technique is introduced, in which $S_{\mathrm{DOB}}(s)$ is to be reduced at low frequencies and $T_{\mathrm{DOB}}(s)$ at high frequencies using corresponding frequency weighting functions of low-pass and high-pass respectively. In the DOB method based on traditional filter model, for example binomial model, cutoff frequency is tuned to achieve the robust stability and the performance of suppressing disturbance and sensor noise. Fig. 1 shows frequency response of a DOB's sensitivity functions in the form of three order binomial coefficient model (solid line) and that of its modified coefficient model

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The authors are with the Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, China (e-mail: yjn2914@yahoo.cn; jbsu@sjtu.edu.cn).

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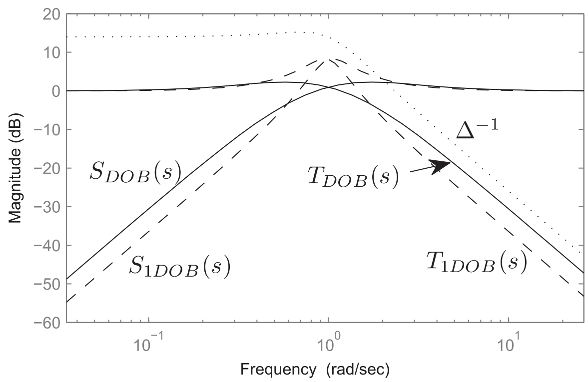

Fig. 1. Sensitivity functions in traditional filter model and its modified model.

(dashed line). It indicates that, in binomial coefficient model, once the cutoff frequency ω_c is fixed ($\omega_c = 1$ rad/s), all performances are settled and can no longer be improved. On the contrary, in modified model, not only cutoff frequency, but also peak magnitude at this frequency are adjusted. These adjustments lead the sensitivity and complementary sensitivity functions lower than the traditional form at both low and high frequencies, respectively. It results in improving the performances of suppressing disturbance and sensor noise under the condition of satisfying robust stability against model perturbation Δ (its inverse represented in dotted line). The above analysis shows that traditional filter model, commonly employed in DOB, does not provide disturbance suppressing performance optimized as possible. In order to achieve optimal disturbance removing performance as well robust stability, systematic method considering all the feedback conditions should de employed.

On the other hand, DOB design methods using H_∞ control theory are widely investigated in recent years. Although the performance of all close systems can be represented in the form of H_∞ inequality, these methods are based on numerical computation norm reduction. The reason why DOB design problem includes special structural restrictions of Q-filter caused by relative order and internal model order specifications. A method of solving DOB design problem in systematic manner is proposed in the next section.

III. Design of DOB Guaranteeing Robust Stability

In application of DOB to the feedback servo control system with parameter variation, two substantial problems should be taken into account: 1) guaranteeing robust stability against model perturbation caused by variations of parameter and load and 2) attaining the optimal performances of suppressing disturbance and sensor noise. Therefore, these design specifications and restrictions should be inflected in the cost function for optimal design.

A. Robust Stability

Q-filters in the DOB, first of all, should be designed to satisfy robust stability condition to model uncertainty. Suppose that model uncertainty can be regarded as a multiplicative perturbation $P(s) = P_n(I + \Delta(s))$, where $P_n(s)$ is nominal plant model and variation $\Delta(s)$ is assumed to be stable. The disturbance observer loop (inner loop) is robustly stable if

$$\overline{\sigma}(\Delta(j\omega) \cdot T_{\text{DOB}}(j\omega)) < 1 \quad \forall\omega \tag{1}$$

where $\overline{\sigma}(.)$ represents maximal singular value. Robust stability condition of DOB loop can not perfectly reflect that of whole closed-loop system, but many reports [9], [14], [15] employed this condition for robust stability. The perfect robust stability condition of closed-loop system to model perturbation is as follows:

$$\overline{\sigma}(\Delta(j\omega) \cdot T(j\omega)) < 1 \quad \forall\omega \tag{2}$$

where

$$T(s) = \frac{P_n(s)C(s) + Q(s)}{1 + P_n(s)C(s)} = \frac{P_n(s)C(n) + T_{\text{DOB}}(s)}{1 + P_n(s)C(s)} \tag{3}$$

is complementary sensitivity function of the closed-loop system and $P_n(s)$ and $C(s)$ are nominal plant and feedback controller separately. If an upper limit function $W_U(s)$ of variation $\Delta(s)$ is chosen such as

$$\overline{\sigma}(\Delta(j\omega)) \leq |W_U(j\omega)| \quad \forall\omega \tag{4}$$

then the sufficient condition of robust stability for a closed-loop system can be written as

$$|T(j\omega)| < |W_U^{-1}(j\omega)| \quad \forall\omega \Leftrightarrow \|W_U(s) \cdot T(s)\|_\infty < 1. \tag{5}$$

B. Low Sensitivity

On the other hand, since the disturbance suppression performance depends on the sensitivity function of the closed-loop system

$$S(s) = \frac{1 - Q(s)}{1 + P_n(s)C(s)} = \frac{S_{\text{DOB}}(s)}{1 + P_n(s)C(s)}. \tag{6}$$

Its gain should be low enough at low frequencies. Thus, the cost function for sensitivity performance can be written as follows:

$$\min \|W_P(s) \cdot S(s)\|_\infty \tag{7}$$

where $W_P(s)$ is weighting function reflecting frequency spectrum of disturbances at low frequencies. In order to design an optimal DOB, we should minimize (7) being subject to (5).

In most researches, this problem have been solved by numerical computation method where Q-filter's coefficients is used as optimal variables [11], [12]. It not only needs to compose complicated matrix inequalities but also is difficult to ensure global optimality and structural order restrictions of Q-filter, such as relative order and internal model order, etc. The most important problem is that the detailed design specifications of DOB as a part of feedback system can not be directly reflected in this problem.

C. Transformation to Design Problem of Standard H_∞ DOB

To solve this optimization problem, some transformations of the sufficient condition of robust stability are considered. (5) can be rewritten as

$$|L(j\omega) + Q(j\omega)| < |W_{UP}^{-1}(1 + L(j\omega))| \quad \forall\omega \tag{8}$$

where $L(s) = P_n(s)C(s)$ is the open loop transfer function of closed-loop system under nominal condition. By the property of absolute value of complex number, we have

$$|L(j\omega) + Q(j\omega)| < |L(j\omega)| + |Q(j\omega)| \quad \forall\omega. \tag{9}$$

Thus, (8) always holds if $Q(s)$ satisfies

$$Q(j\omega)| < |W_{UP}^{-1}(1 + L(j\omega))| - |L(j\omega)| =: E(\omega) \quad \forall\omega. \tag{10}$$

Lemma 1: The frequency function $E(\omega)$ satisfies $E(\omega) > 0$ for all ω if complementary sensitivity function $T_0(s) = L(s)/(1 + L(s))$ without DOB inner loop satisfies nominal stability and general robust stability condition

$$T_0(s) \in RH_\infty, \quad \|W_{UP}(s)T_0(s)\|_\infty < 1. \tag{11}$$

Proof: From (11), we have

$$\left|\frac{L(j\omega)}{1 + L(j\omega)}\right| < |W_{UP}^{-1}(j\omega)| \quad \forall\omega.$$

It is equivalent to

$$|L(j\omega)| < |W_{UP}^{-1}(j\omega)(1 + L(j\omega))|.$$

Thus, we have $E(\omega) > 0\ \forall\omega$ when (11) is satisfied. ■

Let a stable weighting function $W_Q(s)$ be selected such that

$$|W_Q^{-1}(j\omega)| < E(\omega) \quad \forall\omega. \tag{12}$$

From (10) and (12), it comes that the closed-loop system is robustly stable when

$$\|W_Q \cdot T_{\mathrm{DOB}}(s)\|_\infty = \|W_Q \cdot Q(s)\|_\infty < 1. \tag{13}$$

In general, robust stability specification by (13) may be somehow conservative since it is constructed using the right side of (9) instead of the left one. However, in most cases of practice applications, since phases delays of Q(s) and L(s) are nearly similar around cutoff frequency of Q(s) and the frequencies more than that, different between two sides of (9) are small, thereby, conservative of robust stability sufficient condition (11) is also slight.

On the other hand, if a stable weighting function $W_C(s)$ is selected such that

$$|W_P(j\omega)(1 + L(j\omega))^{-1}| \le |W_C(j\omega)| \quad \forall\omega \tag{14}$$

then the optimization problem is

$$\min_{Q(s)} \|W_C(s) \cdot S_{\mathrm{DOB}}(s)\|_\infty$$

subject to

$$\|W_Q(s) \cdot T_{\mathrm{DOB}}(s)\|_\infty < 1$$

or equivalently

$$\max \gamma, \min_{Q(s)} \left\| \begin{bmatrix} \gamma\, W_C(s) \cdot S_{\mathrm{DOB}}(s) \\ W_Q(s) \cdot T_{\mathrm{DOB}}(s) \end{bmatrix} \right\|_\infty < 1 \tag{15}$$

where $Q(s)$ subjects to restriction

$$Q(s) \in \Omega_{n,k,q} = \left\{ \begin{aligned} &R(s) | R(s) = M(s)/N(s) \\ &N(s) = \sum_{i=0}^{n} a_i \cdot s^i, \quad M(s) = \sum_{j=0}^{m} b_j \cdot s^j \\ &n - m = k,\ a_l = b_l\ (l = 0, \ldots, q) \end{aligned} \right\} \tag{16}$$

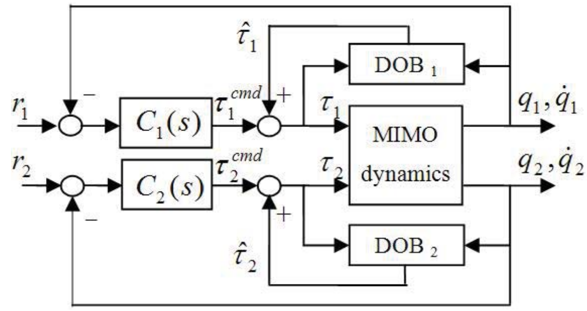

Fig. 2. Decoupling of MIMO dynamics of two-link manipulator by DOB.

and should be selected so that the closed loop system can be stable. n, $k(\le n)$ and $q(\le m)$ are whole order, relative order and internal model order of $Q(s)$ to be obtained in design process, respectively [8]. Notice that the optimization problem (15) subject to (16) is in the form of the standard H_∞ DOB design problem, meaning that it can be systematically solved by the pseudo loop function method proposed in [14]. In this method, the global optimality can easily be achieved using systematic and straightforward procedure without numerical computation with respect to the coefficients of Q-filter.

IV. Application to Control of a Two-Link Manipulator

In this section, the proposed method of designing robust optimal DOB is applied to control for a two-link manipulator with elasticity of the joint shafts. There are some researches that have employed DOB to realize the decentralization of coupled multiple-input, multiple-output (MIMO) dynamics and improved the control performances of two-link manipulators [2], [7]. In these researches, by experimentally tuning of the cutoff frequency of Q-filter in DOB, trade-off between disturbance suppressing performance and sensor noise removing performance is realized. However, in many practices of industrial robots, actuators have torsional shafts and increasing of cut-off frequency of the Q-filter is restricted by modeling error caused by the elasticity and parameter variations, etc. The researches on the method for improving the disturbance suppressing performance under the restricted cut-off frequency seldom appear in the research area of DOB. In the presence of both the coupled link dynamics and model uncertainty of joint actuators, to verify the performances of the proposed method, a two-link manipulator is used as a plant in this section. The proposed design method of DOB can be used to design optimal robust DOB guaranteeing the robust stability against the unmodeled dynamics and variations of parameters, and achieving optimal disturbance suppressing performance.

A. Decentralization by DOB

The dynamics of a vertical two-link manipulator can be described as

$$J(q)\ddot{q} + B\dot{q} + C(q, \dot{q}) + G(q) = \tau \tag{17}$$

where $J(q)$, B, $C(\dot{q})$, and $G(q)$ are inertia matrix, diagonal matrix of viscous friction coefficients, centrifugal and coriolis torque vector and gravity force vector of corresponding

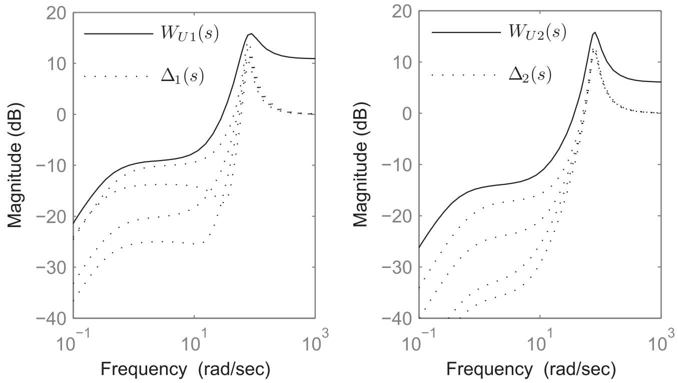

Fig. 3. Model perturbations and their upper limit functions.

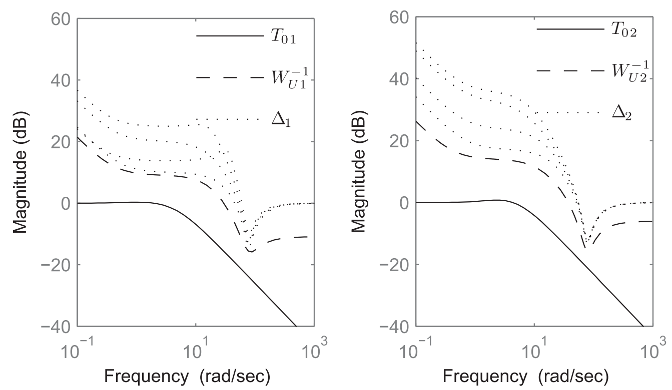

Fig. 4. Test of robust stability condition without DOB.

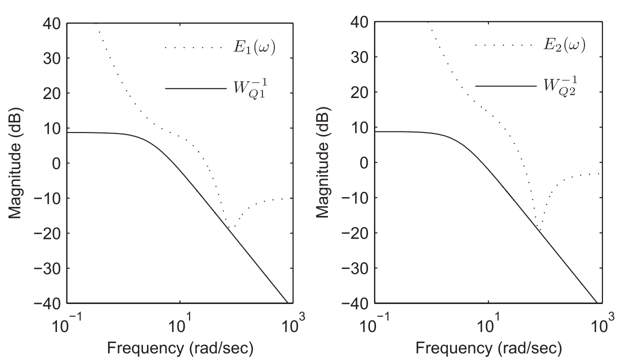

Fig. 5. Verifications of (12) for each joint.

orders, separately. Each matrix and their parameters values are shown in Appendix A. q and τ are the vectors of joint angle and driving torque. DOB can decentralize the coupled MIMO dynamics of (17) into individual decoupled single-input, single-output (SISO) servo systems as shown in Fig. 2 [7]

$$J_n\ddot{q} + B_n\dot{q} = \tau - \tau_d = \tau^{cmd} + \hat{\tau}_d - \tau_d \tag{18}$$

where J_n and B_n are diagonal nominal inertia matrix and diagonal nominal viscous coefficient matrix. From (18), it is clear that if driving torque τ consists of command torque τ^{cmd} and estimation disturbance $\hat{\tau}_d$, it can cancel the real disturbance τ_d, then MIMO system (17) could be transformed into decoupled SISO servo systems.

B. Design of Optimal Robust DOB

Performance of the decentralization highly depends on Q-filter of DOB. To achieve better performance, it needs to enlarge its pass band by increasing the cut-off frequency. However, the cut-off frequency is restricted by sensor noise and modeling error at high frequencies. To improve control performances under the condition of restricted cut-off frequency, more sophisticated design of DOB is needed.

In many practices, modeling errors at high frequencies occur in actuator model. Especially, elasticity shaft of actuators which is modeled as two-inertia system causes the modeling error at much higher frequencies than that of link dynamics. Therefore, the cut-off frequency is mainly restricted by actuator's shaft oscillation characteristic. Considering the flexibility, the dynamics of (17) can be modified as

$$\begin{cases} J_M\ddot{q}_M + B_M\dot{q}_M = \tau_M - K_f(q_M - q) \\ J(q)\ddot{q} + B\dot{q} + C(q,\dot{q}) + G(q) = \tau = K_f(q_M - q) \end{cases} \tag{19}$$

where J_M, B_M, and K_f are the inertia moment of the motor, viscous friction coefficient and elasticity coefficient of shaft, respectively. q_M and τ_M are respectively the driving angle and the driving torque of the motor. From (19), if we neglect the speed variety of link dynamics (in general, link dynamics speed is much lower than actuator's shaft dynamics), the transfer function $G_{q\,\tau_M}(s)$ from τ_M to q is

$$G_{q\,\tau_M}(s) = \frac{1}{s}\cdot\frac{b_{G0}}{a_{G3}s^3 + a_{G2}s^2 + a_{G1}s + a_{G0}} \tag{20}$$

where $a_{G3} = J_M J$, $a_{G2} = J_M B_n + J B_M$, $a_{G1} = K_f(J_M + J) + B_M B$, $a_{G0} = b_{G0} = K_f(B_M + B)$, and the nonlinear terms are regarded as disturbance to be compensated by DOB.

The parameters of both joints are shown in Appendix C. The real load inertia J varies around its nominal value J_n. If transfer function model $G_n(s)$ from τ^{cmd} to q in (18) is used as a nominal model of DOB, then relative model perturbation $\Delta(s) = (G_{q\,\tau_M}(s) - G_n(s))/G_n(s)$ should be taken into account for robust stability. Fig. 3 shows the frequency responses of $\Delta(s)$ for the different values of load inertia J in motion scope. From this figure, it is clear that functions

$$W_{U1}(s) = \frac{3.5s(s^2 + 50s + 600)}{(s + 0.4)(s^2 + 50s + 6000)} \tag{21}$$

$$W_{U2}(s) = \frac{2s(s^2 + 60s + 600)}{(s + 0.4)(s^2 + 30s + 6000)} \tag{22}$$

can be used as upper limit functions for $\Delta(s)$ due to parameter variations. If the feedback controllers of outer loop is selected as PD compensators shown in Appendix B, the closed-loops without DOB satisfy the general robust stability (11) as shown in Fig 4. Considering nominal plant's relative order $k = 1$, internal model order $q = 2$ and whole order $n = 3$ of Q-filter, the weighting functions satisfying (10) can be selected as

$$W_{Q1}(s) = \frac{1}{8.2}(s + 3.0), \quad W_{Q2}(s) = \frac{1}{6.2}(s + 2.2). \tag{23}$$

Fig. 5 shows that weighting functions $W_{Q1}(s)$ for the first joint and $W_{Q2}(s)$ for the second joint satisfy the inequality

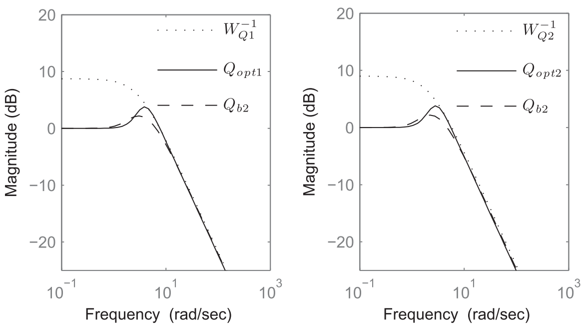

Fig. 6. Frequency responses of Q-filters for each joint.

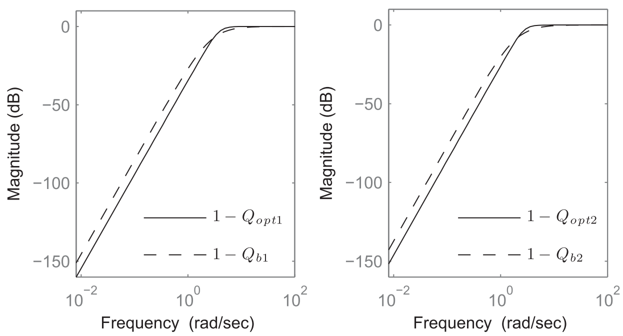

Fig. 7. Comparisons of sensitivity functions.

conditions of (12). Thus (23) can be used as weighting functions for robust stability. Considering the maximal value $q = 2$ of the internal model order [8], the low sensitivity weighting functions $W_{C1}(s)$ and $W_{C2}(s)$ can be simply selected as

$$W_{C1}(s) = W_{C2}(s) = \frac{1}{(s+\lambda)^3} \tag{24}$$

where $\lambda > 0$ is a sufficiently small number to attain internal model order specification [14]. For these weighting functions, design problem (15) and (16) for Q-filters of both joints are systematically solved by standard H_∞ DOB design method

$$Q_{\text{opt1}}(s) = \frac{8.2s^2 + 28.8476s + 48.2734}{s^3 + 8.2s^2 + 28.8476s + 48.2734} \tag{25}$$

$$Q_{\text{opt2}}(s) = \frac{6.2s^2 + 16.6408s + 21.2412}{s^3 + 6.2s^2 + 16.6408s + 21.2412} \tag{26}$$

The frequency responses of the designed Q-filters are shown in Fig. 6, as well as their weighting functions. From this figure, it can be known that the magnitude differences between the Q-filters $Q_{\text{opt},i}(s), i = 1, 2$ by proposed method and $Q_{b,i}(s), i = 1, 2$ of binomial model around cut-off frequency are considerably large, coinciding the results from weighting functions of (23) that allow different peak magnitudes around this frequency. Because of these differences of magnitude around cut-off frequency, the differences of sensitivity functions between $1 - Q_{\text{opt}}(s)$ and $1 - Q_b(s)$ at low frequencies are also significantly large (about 10 dB or 3 times) as shown in Fig. 7, resulting in considerably large differences between performances of suppressing disturbance. Fig. 8 shows that four Q-filters entirely satisfy the robust stability condition (5) with DOB.

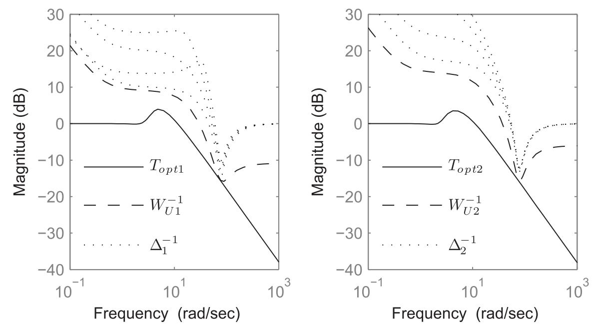

Fig. 8. Test of robust stability condition (5) with DOB for every joint.

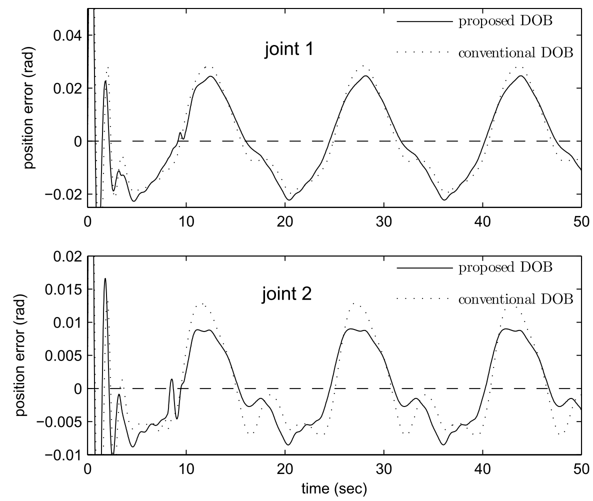

Fig. 9. Error responses to sinusoidal references.

C. Simulation Analysis

The control system for the two link manipulator (19) is constructed using PD compensators shown in Appendix B as feedback controllers $C(s)$ and the Q-filters shown in (25) and (26) for every joint, according to the block diagram in Fig. 2.

Fig. 9 shows the error responses to sinusoidal references of magnitude 1 rad and frequency 0.4 rad/s for every joint. From the figure, it is verified that error amplitude of responses by DOB with $Q_{\text{opt}}(s)$ (shown in solid line) are smaller than that of conventional model $Q_b(s)$ (shown in dotted line).

The substantial differences between the proposed method and the conventional one in disturbance suppression performance are compared. Fig. 10 shows the error responses to sinusoidal disturbance of amplitude 1 Nm and frequency of 0.5 rad/s for both joints. Errors by the proposed DOB (solid line) are about 3 times smaller than that by the conventional DOB (dotted line), and these simulation results entirely coincide with the design results shown in Fig. 7.

Fig. 11 shows the error responses of the two joints to step disturbances of 1 Nm acting at the moment of $t = 20$ s. The errors by conventional DOB show maximum deviation of 0.0095 rad from equilibrium position for the first joint and 0.0053 rad for the second joint, while the errors by proposed DOB show 0.003 rad for the first joint and 0.0026 rad for the second joint. Performance of suppressing disturbance by the proposed DOB is extremely

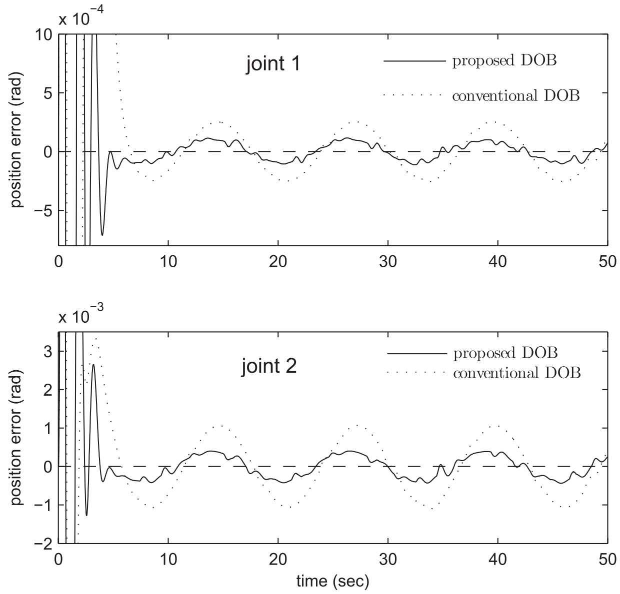

Fig. 10. Error responses to sinusoidal disturbance.

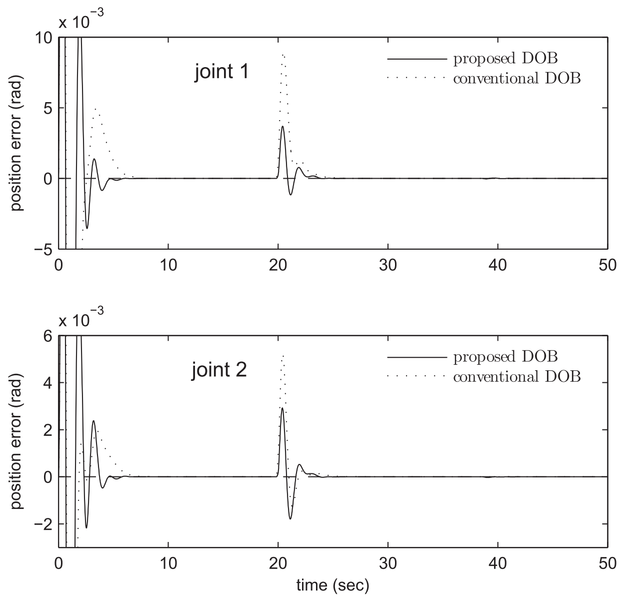

Fig. 11. Error responses to step disturbances.

useful for controlling the robot under various types of exotic disturbances.

The above experimental results show that the proposed design method of Q-filter of DOB can achieve optimal disturbance suppressing performance under the condition of guaranteeing robust stability against model perturbation caused by variations of parameters and load.

V. Conclusion

This brief presented a design method of the optimal DOB satisfying robust stability against model perturbation caused by parameter variation. The cutoff frequency of Q-filter played an important role in improving the performance of suppressing disturbance, but it was restricted by model perturbation for robust stability. Using the Q-filter design method proposed in this brief, a robust DOB that provides optimal disturbance suppression performance can be obtained by systematic optimization of cost functions, without tuning cutoff frequency. In the motion control plants such as robot manipulators whose joints include mechanical torsional shafts, the proposed DOB can decentralize the interactions between links and compensate for the nonlinearity by the disturbance suppression property. Simulation results in control of vertical two-link manipulator with flexible joints show the effectiveness of this method.

TABLE A-I
Parameters of the Robot Dynamics

	Link 1	
l_1	Length of the link (m)	1
l_{C1}	Distance from the joint axis to the center of mass (m)	0.5
m_1	Mass of link (kg)	50
I_{C1}	Moment of inertia with respect to the center of mass $(kg \cdot m^2)$	10
m_{m1}	Mass of the rotor of the motor (kg)	5
I_{m1}	Moment of inertia with respect to rotor axis $(kg \cdot m^2)$	0.02
k_1	Gear ratio of the motor	100
	Link 2	
l_2	Length of the link (m)	1
l_{C2}	Distance from the joint axis to the center of mass (m)	0.5
m_2	Mass of link (kg)	50
I_{C2}	Moment of inertia with respect to the center of mass $(kg \cdot m^2)$	10
m_{m2}	Mass of the rotor of the motor (kg)	5
I_{m2}	Moment of inertia with respect to rotor axis $(kg \cdot m^2)$	0.02
k_2	Gear ratio of the motor	100
g	Acceleration of gravity $(kg \cdot m/s^2)$	9.8

Appendix A
Dynamic Model of Two-Link Manipulator

The coefficient matrices and parameters of dynamics model (19) of vertical two-link manipulator used in simulation is as follows:

$$
\begin{aligned}
J(q) &= \begin{bmatrix} J_{11}(q) & J_{12}(q) \\ J_{21}(q) & J_{22}(q) \end{bmatrix} \\
J_{11}(q) &= I_{C1} + m_{C1} l_{C1}^2 + k_1^2 I_{m1} + I_{C2} \\
&\quad + m_{C2}(l_1 + l_{C2} + 2 l_1 l_{C2} \cos q_2) \\
&\quad + I_{m2} + m_{m2} l_1^2 \\
J_{12}(q) &= J_{21}(q) \\
&= I_{C2} + m_{C2}(l_{C2}^2 + l_1 l_{C2} \cos q_2) + k_2 I_{m2} \\
J_{21}(q) &= I_{C2} + m_{C2} l_{C2}^2 + k_2^2 I_{m2} \\
C(q, \dot{q}) &= \begin{bmatrix} m_{C2} l_1 l_{C2}(-2 \sin q_2 \dot{q}_1 \dot{q}_2 - \sin q_2 \dot{q}_2^2) \\ m_{C2} l_1 l_{C2} \sin q_2 \dot{q}_1^2 \end{bmatrix} \\
G(q) &= \begin{bmatrix} G_1(q) \\ G_2(q) \end{bmatrix} \\
G_1(q) &= (m_{C1} l_{C1} + m_{m2} l_1 + m_{C2} l_1) g \cos q_1 \\
&\quad + m_{C2} l_{C2} g \cos(q_1 + q_2) \\
G_2(q) &= m_{C2} l_{C2} g \cos(q_1 + q_2).
\end{aligned}
$$

Refer to Table A-1.

Appendix B
Parameters of the Feedback Controllers

Refer to Table B-1.

TABLE B-I
GAINS OF THE PD COMPENSATORS FOR BOTH JOINTS

Joint 1			Joint 2		
K_{P1}	Proportional gain	1200	K_{P2}	Proportional gain	1800
K_{D1}	Differential gain	1000	K_{D2}	Differential gain	900

TABLE C-I
PARAMETERS OF THE ROBOT JOINTS

	Joint 1	
K_{f1}	Stiffness coefficient of the shaft $(N \cdot m/rad)$	12000
B_1	Viscous friction coefficient of joint $(N \cdot m \cdot s/rad)$	40
B_{M1}	Viscous friction coefficient of the rotor $(N \cdot m \cdot s/rad)$	40
J_{M1}	Inertia moment of the rotor converted to joint axis $(kg \cdot m^2)$	100
J_{n1}	Nominal moment of inertia with respect to the joint $kg \cdot m^2$	200
	Joint 2	
K_{f2}	Stiffness coefficient of the shaft $(N \cdot m/rad)$	12000
B_2	Viscous friction coefficient of joint $(N \cdot m \cdot s/rad)$	40
B_{M2}	Viscous friction coefficient of the rotor $(N \cdot m \cdot s/rad)$	40
J_{M2}	Inertia moment of the rotor converted to joint axis $(kg \cdot m^2)$	100
J_{n2}	Nominal moment of inertia with respect to the joint $(kg \cdot m^2)$	130

APPENDIX C
PARAMETERS OF THE JOINTS

Refer to Table C-1.

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