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Disturbance rejection control for non-minimum phase systems with optimal disturbance observer

Lu Wang, Jianbo Su*

Department of Automation; Key Laboratory of System Control and Information Processing, Ministry of Education, Shanghai Jiao Tong University, Shanghai 200240, PR China

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ABSTRACT

This paper investigates the disturbance rejection control for stable non-minimum phase (NMP) systems with time delay. A robust disturbance observer (DOB) based control structure is proposed. Specifically, the robust DOB is employed to compensate the uncertain plant into a nominal one, based on which a prefilter is adopted to acquire desired performance. Then, a novel DOB configuration strategy for stable NMP systems is proposed. This strategy synthesizes the internal and robust stability, relative order and mixed sensitivity design requirements together to establish the optimization function. The optimal solution is obtained by standard H_∞ theory under the condition of guarantying the presented requirements. We also investigate how the DOB can compensate the uncertain plant into a nominal one. The specific design procedure is presented for an uncertain plant with both unstable zeros and time delay.

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1. Introduction

In most practical industrial processes, the inevitable system uncertainties and external disturbances will have great influence on the performance of control system [1]. The efficient disturbance rejection method is the compensation by the estimation of model mismatch and external disturbances [2–5]. The disturbance observer (DOB) based control method was originally proposed by Ohnishi in 1987 [6], whose effectiveness in disturbance rejections has been shown in many applications, such as humanoid joint control [7], robot manipulators control [8], aircraft control [9], optical disk control [10], motor control [11], vibration control [12,13], ball mill grinding circuits control [14,15], etc.

DOB consists of a nominal model and a low-pass filter named Q filter. The design of Q filter is the key point of DOB configuration, which attracts many researchers' attentions. Typical filter forms, such as the binomial coefficient and Butterworth filter [16,17] are widely used. At this point, the performance mainly depends on the cut-off frequency. However, selection of parameters mainly depends on experiences, while there is no specific criterion for evaluation. The performance of system is largely limited by the fixed form of Q filter. Moreover, the robust stability can only be analyzed after the Q filter is determined. This means that we

should repeat the design procedure until the robust stability requirement is guaranteed. There have been abundant results in using the H_∞ theory for Q filter design [18–20]. Linear Matrix Inequality or Algebraic Riccati Equation is applied in [18,19] to optimize the Q filter with static gain. The standard H_∞ theory is employed in [20] to optimize the Q filter.

However, these researches neglect the internal stability of the system, and these methods cannot be used directly in non-minimum phase (NMP) systems. Since the inverse of nominal plant is required in DOB configuration, the internal stability problem occurs if the nominal plant has the right-half of the s -plane (RHP) zeros. Meanwhile, the inverse of nominal model is non-causal according to the time delay. The configuration of DOB based control system for NMP system has been widely concerned in recent years. The authors present a DOB based model predictive control (MPC) method for a NMP process with time delay [14,15]. However, RHP zeros of the plant are not considered. In [21,22], a new filter in parallel with the Q filter is designed, and hence the parallel connection with the plant becomes of minimum phase. Then the conventional DOB configuration procedure is employed for the new system. A non-causal, minimum phase transfer function is proposed in [23] to remove the internal stability problem caused by RHP zeros. Nevertheless, the time delay is not considered in these methods. The improved DOB-MPC scheme is proposed in [24] for stable NMP systems, in which both RHP zeros and time delay are taken into account. An allpass portion is introduced in the Q filter to eliminate the influence caused by the

* Corresponding author.

E-mail address: jbsu@sjtu.edu.cn (J. Su).

NMP property of the plant. However, both internal and robust stability of the system are not considered and the performance of the DOB is not analyzed specifically. In summary, although there are several researches in DOB configuration for NMP system, there is no systematic design of DOB for such system with both RHP zeros and time delay.

In this paper, we focus on the DOB based controller design for NMP systems. A novel DOB configuration strategy is proposed systematically. We first consider the robust internal stability, relative order, mixed sensitivity design requirements together to establish an optimization function. Then, the optimization problem is transformed into a standard H_∞ one, based on which the solution of Q filter is optimized by the existing standard H_∞ theory. We also investigate how the DOB can compensate the real plant into the nominal one. At last, a design example is presented specifically on an uncertain plant with both RHP zeros and time delay. In summary, the proposed strategy can be successfully employed in the DOB configuration for stable NMP systems, and the main contributions of this paper are presented as follows:

- (1) A systematic design strategy of DOB is proposed for stable NMP systems by taken both RHP zeros and time delay into account.
- (2) The optimization function is established by taken robust internal stability, relative order, mixed sensitivity design requirements into account together.
- (3) The solution of Q filter is optimized by introducing the allpass portion into virtually controlled objective of H_∞ problem, based on which the unstable poles of inverse of nominal plant can be eliminated for robust internal stability constraints.
- (4) Discussions on how the DOB can compensate the real plant into the nominal one are presented, which are verified in the simulations.

The rest of this paper is organized as follows. In Section 2, the control structure of the system is analyzed and the NMP problem is stated. In Section 3, we give the constraints of controller and DOB using internal stability principle, then the DOB is designed to acquire desired requirements. The performance of the proposed DOB is also under exploration. In Section 4, we consider a stable NMP plant with RHP zeros and time delay to implement the design procedure specifically. In Section 5, simulation on a rotary mechanical system is carried out to show the effectiveness of the proposed method, followed by Conclusions in Section 6.

2. Problem formulation

The traditional control system based on DOB is expressed in Fig. 1, where $P(s)$ is the plant model, $P_n(s)$ is the nominal model, $C(s)$ and $Q(s)$ are the controller and Q filter to be designed. $R(s), Y(s), D(s)$ and $N(s)$ denote the Laplace transformation of reference input r , output y , external disturbances d and measurement noise n , respectively. τ is the dead-time of the plant while τ_n is its nominal value. DOB considers the mismatch between real plant and nominal one as equivalent disturbance acting on its nominal model. It can estimate the equivalent disturbances

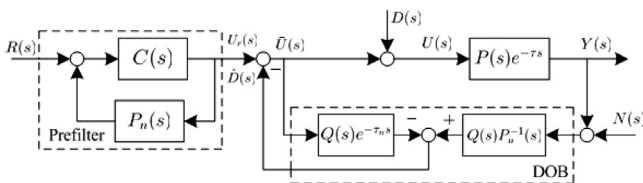


Fig. 1. The DOB based controller for the NMP system.

combined with the external disturbances, and feed back the estimated disturbances for cancellation. Then the controller is applied as a prefilter to stabilize the nominal system compensated by DOB for desired performance. This control system has two degrees-of-freedom (2DOF). Hence, the DOB can be optimized for disturbance rejection performance and then the prefilter can be chosen independently for setpoint tracking performance. In this control system, the design and optimization of controller and DOB can be implemented separately.

The control objective is to design the DOB to attenuate the compound disturbance caused by model mismatch and external disturbances. Then, a prefilter is designed to stabilize the nominal model to acquire desired performance. Since DOB can estimate the compound disturbances, the controller $C(s)$ can be designed based on the nominal plant $P_n(s)$. Here, H_2 theory is applied to design the controller $C(s)$ and the prefilter can be obtain as:

$$\frac{C(s)}{1 + C(s)P_n(s)} \tag{1}$$

For the NMP systems, the inverse of the RHP zeros and time delay is physically unrealizable for its non-causal property. The internal stability is a basic requirement for a practical closed-loop system, but unfortunately, in the research work of DOB design, the internal stability analysis for this kind of system has been only discussed in [25]. Moreover, most DOB design methods only validate the robust stability after the parameters of the DOB are determined. It is a complicated work for us to adjust the parameters repeatedly to guarantee the robust stability. Thus, in this paper, we mainly focus on the systematic design strategy of DOB for stable NMP systems. The system model $P(s)$ is described with multiplicative uncertainty as:

$$P(s) = P_n(s)(1 + \Delta(s)), \tag{2}$$

where $P(s)$ and $P_n(s)$ are all stable plants with NMP property. The nominal model $P_n(s)$ is expressed as:

$$P_n(s) = \frac{N(s)}{D(s)} \prod_i (-s + \xi_i), \tag{3}$$

where $Re(\xi_i) > 0$, $N(s)$ and $D(s)$ have no root with positive real part.

3. Robust DOB design

The transfer function of the DOB structure can be written as:

$$Y(s) = M^{-1}(s)[P(s)e^{-\tau s}(1 - Q(s)e^{-\tau_n s})D(s) + P(s)e^{-\tau s}U_r(s) - P_n^{-1}(s)P(s)Q(s)e^{-\tau s}N(s)]. \tag{4}$$

where $M(s) = 1 + P_n^{-1}(s)P(s)Q(s)e^{-\tau s} - Q(s)e^{-\tau_n s}$.

From Eq. (4), we notice that the suppression performance against external disturbances d mainly depends on the factor $(1 - Q(s)e^{-\tau_n s})$. However, it is very hard for us to optimize $Q(s)$ directly with time delay. By introduce the notation $Q'(s) \triangleq Q(s)e^{-\tau_n s}$, we can consider $Q'(s)$ directly. Then, $Q(s)$ can be obtained by replace the time delay $e^{-\tau_n s}$ with a Padé approximation. Eq. (4) can be rewritten as:

$$Y(s) = M^{-1}(s)[P(s)e^{-\tau s}(1 - Q'(s))D(s) + P(s)e^{-\tau s}U_r(s) - P_n^{-1}(s)P(s)Q'(s)N(s)]. \tag{5}$$

Assume that nominal model of the plant matches the real plant very well (i.e., $P(s) = P_n(s)$, $\tau = \tau_n$), then Eq. (5) can be simplified as:

$$Y(s) = P_n(s)e^{-\tau_n s}(1 - Q'(s))D(s) + P_n(s)e^{-\tau_n s}U_r(s) - Q'(s)N(s). \tag{6}$$

From Eq. (6), we can see that $Q'(s)$ should be reduced as far as possible to attenuate the influence caused by measurement, whereas $1 - Q'(s)$ should also be small to reject the external disturbances. These two conditions are conflicting. In practical

applications, disturbances normally happen at low frequencies, whereas measurement noise and system uncertainties take effects at high frequencies. This suggests that $Q'(s)$ should be designed as a low-pass filter, which can be designed by frequency weighted minimization.

3.1. Robust internal stability

A control system is internally stable if bounded signals injected at any point of the control system generate bounded responses at any other point [26]. The internal stability is a basic requirement for a practical closed-loop system. Here, we give the sufficient condition of the internal stability of the DOB based control structure.

Theorem 1. *The control system described in Fig. 1 is internally stable if the following requirements are satisfied:*

- Requirement 1. $\frac{C(s)}{1+P_n(s)C(s)}$ has no RHP poles;
- Requirement 2. $Q(s) \in \mathcal{RH}_\infty$ and it can eliminate all the RHP poles of $P_n^{-1}(s)$.
- Requirement 3. The open loop transfer function from U_r to U is stable.
- Requirement 4. The robust stability of the closed-loop system against the system uncertainties should be satisfied.

Proof. We can easily find that the transfer function from $R(s)$ to $U_r(s)$ is $\frac{C(s)}{1+P_n(s)C(s)}$, hence Requirement 1 must be satisfied.

Then we analyze the internal stability of the DOB structure. With the configuration of Fig. 1, the six transfer functions from $[U_r \ D \ N]^T$ to $[Y \ U]^T$ are given as:

$$\frac{1}{M(s)} \begin{bmatrix} P(s)e^{-\tau s} & P(s)e^{-\tau s}(1-Q(s)e^{-\tau_n s}) & -P_n^{-1}(s)P(s)Q(s)e^{-\tau s} \\ 1 & (1-Q(s)e^{-\tau_n s}) & -P_n^{-1}(s)Q(s) \end{bmatrix}, \quad (7)$$

If all the components of the matrix in Eq. (7) and $\frac{1}{M(s)}$ are all in \mathcal{RH}_∞ , then the transfer function from $[U_r \ D \ N]^T$ to $[Y \ U]^T$ is stable, based on which we can guarantee the internal stability.

Since $P(s)$ and $P_n(s)$ are with no RHP poles, if **Requirement 2** is satisfied, then we can find that all components of matrix in Eq. (7) are in \mathcal{RH}_∞ .

Noticing that $\frac{1}{M(s)}$ is the transfer function from U_r to U , if open loop transfer function from U_r to U is stable and the closed-loop system is robustly stable, then the close-loop system is input output stable with input U_r and output U . Hence, $\frac{1}{M(s)} \in \mathcal{RH}_\infty$ if Requirement 3 and 4 are satisfied. Finally, we can come to a conclusion that the control system described in Fig. 1 is internally stable if Requirements 1 to 4 are satisfied. \square

Theorem 2. *If the following inequality is satisfied, the condition of robust stability described in Requirement 4 of Theorem 1 is satisfied.*

$$|Q'(j\omega)W_\Delta(j\omega)| < 1, \quad \forall \omega, \quad (8)$$

where $W_\Delta(j\omega) > F(\omega)$, $\forall \omega$, and the definition of $F(\omega)$ is given as

$$F(\omega) = |\Delta(j\omega)e^{-(\tau-\tau_n)j\omega}| + |1 - e^{-(\tau-\tau_n)j\omega}|, \quad \forall \omega \quad (9)$$

Proof. The equivalent structure is given in Fig. 2. According to the Small Gain Theory, we get the requirement of robust stability as:

$$\left\| \frac{Q(s)e^{-\tau s}}{Q(s)e^{-\tau_n s} - Q(s)e^{-\tau s} - 1} \Delta(s) \right\|_\infty < 1. \quad (10)$$

The triangle inequality is applied to acquire a more simple expression. According to Eq. (10), we get:

$$|Q'(j\omega)\Delta(j\omega)e^{-(\tau-\tau_n)j\omega}| < |Q'(j\omega)(1 - e^{-(\tau-\tau_n)j\omega}) - 1|, \quad \forall \omega. \quad (11)$$

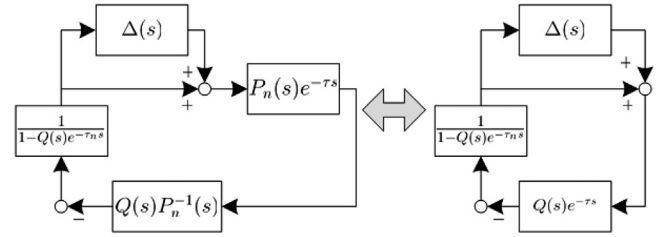


Fig. 2. Equivalent structure of robust stability analysis.

From the triangle inequality, we get:

$$|Q'(j\omega)(1 - e^{-(\tau-\tau_n)j\omega}) - 1| \geq |1 - Q'(j\omega)(1 - e^{-(\tau-\tau_n)j\omega})|, \quad \forall \omega. \quad (12)$$

Then, if the following Eq. is satisfied, then Eq. (11) is satisfied.

$$|Q'(j\omega)\Delta(j\omega)e^{-(\tau-\tau_n)j\omega}| + |Q'(j\omega)(1 - e^{-(\tau-\tau_n)j\omega})| < 1, \quad \forall \omega. \quad (13)$$

With the definition of $F(\omega)$, we can easily obtain that the closed-loop system is robustly stable against the system uncertainties if Eq. (8) holds. \square

Remark 1. The above two Theorems provide us with the robust internal stability constraints for DOB configuration. These above constraints will be applied to establish the cost function.

3.2. Establishment of the cost function

The sensitivity and complementary sensitivity function of the DOB structure are the transfer functions from equivalent disturbance at output terminal and input u_r to output y , respectively. From Fig. 1, we can derive:

$$S_{DOB} = \frac{1-Q'(s)}{M(s)}, \quad T_{DOB} = \frac{P_n^{-1}(s)P(s)Q'(s)}{M(s)}. \quad (14)$$

With the assumption that nominal model of the plant matches the real plant very well, This results in $T_{DOB}(s) = Q'(s)$ and $S_{DOB}(s) = 1 - Q'(s)$. Then, it is shown that in order to achieve disturbance rejection, robustness against system uncertainties and noise attenuation performance, it is desirable to reduce $Q'(s)$ and $1 - Q'(s)$ as much as possible. Since external disturbances is normally at low frequency while measurement noise is at high frequencies, $1 - Q'(s)$ should be minimal in low frequencies (control band), whereas $Q'(s)$ should be minimal in high frequencies (system uncertainties and noise band).

From the descriptions above, we know that the design requirements of Q filter can be summarized as follows:

1. The relative order restrictions of $Q(s)$ should higher than or at least equal to that of $P_n(s)$ to make sure that $Q(s)P_n^{-1}(s)$ is a proper transfer function.
2. The designed Q filter should eliminate the compound disturbance as well as the influence caused by measurement noise.
3. The robust stability of the closed-loop system should be satisfied.

We define a set of $Q'(s)$ as:

$$\Omega_k = \left\{ F(s) | F(s) = \frac{A(s)}{B(s)} = \frac{\sum_{j=0}^q a_j s^j}{\sum_{i=0}^p b_i s^i}, \quad a_q \neq 0, \quad b_p \neq 0, \quad p - q \geq k \right\}, \quad (15)$$

where k is the relative order of $P_n(s)$, $A(s)$ and $B(s)$ are coprime polynomials.

The solution of the Q filter should eliminate the disturbances as far as possible under the condition of guarantying the requirements of internal and robust stability, relative order and

measurement noise suppression performance. Then, we establish the optimization function from the above requirements.

From Eq. (5), the disturbance and measurement noise attenuation problem can be regarded as the selection of the tradeoff between sensitivity and complementary sensitivity functions:

$$\begin{cases} \min_{Q'(s)} |W_1(j\omega)(1 - Q'(j\omega))| \\ \min_{Q'(s)} |W_N(j\omega)Q'(j\omega)|, \end{cases} \quad (16)$$

where W_1 and W_N are the weighting functions which reflect the prior frequency property of the external disturbances and measurement noise, respectively. Here, we can transform the above functions into the following norm:

$$\begin{cases} \max_{Q'(s)} \gamma, \min_{Q'(s)} \|\gamma W_1(s)(1 - Q'(s))\|_\infty < 1 \\ \|W_N(s)Q'(s)\|_\infty < 1. \end{cases} \quad (17)$$

With the consideration of the robust internal stability of closed-loop system, we combine the requirements in Eqs. (8) and (17) together to acquire the new optimization function as:

$$\begin{aligned} \max \quad & \gamma, \\ \text{s.t.} \quad & \min_{\substack{Q'(s) \in \Omega_k \\ Q'(s) \in RH_\infty}} \left\| \begin{bmatrix} \gamma W_1(s)(1 - Q'(s)) \\ W_2(s)Q'(s) \end{bmatrix} \right\|_\infty < 1, \end{aligned} \quad (18)$$

where $|W_2(j\omega)| = \max\{|W_\Delta(j\omega)|, |W_N(j\omega)|\}, \forall \omega$. $|W_2(j\omega)|$ should be selected as close as possible to $|W_\Delta(j\omega)|$ and $|W_N(j\omega)|$, or it will lead the system more conservative.

3.3. Q filter optimization

For the optimization problem in Eq. (18), it is very hard to get the solution $Q'(s)$ to guarantee the internal stability requirements. The standard H_∞ theory is applied for this kind of optimization problem in [20]. By define the transfer function of virtual loop as:

$$\tilde{L}(s) = \frac{Q'(s)}{1 - Q'(s)} = \tilde{P}(s)\tilde{K}(s), \quad (19)$$

the Q filter design problem becomes a standard H_∞ problem as:

$$\begin{aligned} \max \quad & \gamma, \\ \text{s.t.} \quad & \min_{\substack{Q'(s) \in \Omega_k \\ Q'(s) \in RH_\infty}} \left\| \begin{bmatrix} \gamma W_1(s)(I + \tilde{P}\tilde{K})^{-1} \\ W_2(s)\tilde{P}\tilde{K}(I + \tilde{P}\tilde{K})^{-1} \end{bmatrix} \right\|_\infty < 1, \end{aligned} \quad (20)$$

where $\tilde{L}(s) = \tilde{P}(s)\tilde{K}(s)$ and $\tilde{P}(s), \tilde{K}(s)$ are the virtually controlled objective and controller, respectively.

The virtually controlled objective $\tilde{P}(s)$ is given as:

$$\tilde{P}(s) = P_0(s)P_A(s), \quad (21)$$

where $P_0(s)$ is a stable plant and $W_2(s)P_0(s)$ must be proper to guarantee the solution of the standard H_∞ problem, $P_A(s)$ is an allpass portion which includes all the RHP zeros of $P_n(s)$:

$$P_A(s) = \prod_i \frac{-s + \xi_i^H}{s + \xi_i^H}, \quad \text{Re}(\xi_i) > 0, \quad (22)$$

where the superscript H denotes complex conjugate.

Theorem 3. The optimal loop function $\tilde{L}^*(s)$ is independent of the selection of $\tilde{P}(s)$ and uniquely determined. In addition, for any $\tilde{P}(s) \in \Sigma_p$, these two H_∞ norm optimization problems in Eqs. (18) and (20) are equivalent.

Proof. Let Σ_p be a set of virtually controlled objectives. For a virtually controlled objective $\tilde{P}(s) \in \Sigma_p$, assume that $\tilde{K}(s)$ is the optimal solution of (20). Then $\tilde{P}(s)$ has k stable poles $p_i^*(i = 1, \dots, k)$. From loop shaping theory, it is clear that the optimal solution $\tilde{K}^*(s)$

of the problem (20) includes k zeros equal to these poles as

$$\tilde{K}(s) = \tilde{K}_0(s) \prod_{i=1}^k (s + p_i^*). \quad (23)$$

Since $\tilde{L}(s) = \tilde{P}(s)\tilde{K}(s) = \tilde{K}_0(s)$ is optimal open-loop function. For another virtual plant, we can get the similar conclusion and $\tilde{K}_0(s)$ is also an optimal open-loop function for $\tilde{P}_1(s)$. This means that selection of $\tilde{P}(s)$ does not have any influence on minimizing the norm of the problem (17), resulting in $\tilde{L}(s) = \tilde{K}_0(s)$. \square

For a given virtually controlled objective $\tilde{P}(s)$, if we can acquire the optimal solution of the virtual controller $\tilde{K}(s)$, then we can get the Q filter as:

$$Q'(s) = \frac{\tilde{P}(s)\tilde{K}(s)}{1 + \tilde{P}(s)\tilde{K}(s)}. \quad (24)$$

Then, due to the definition of $Q'(s)$, we get $Q(s) = Q'(s)e^{\tau_n s}$. Here, a first order Padè approximation is introduced as $e^{\tau_n s} \approx \frac{\mu\tau_n s + 1}{-(1 - \mu)\tau_n s + 1}, \mu \in [0, 1]$, here we simply choose $\mu = 0.5$ and get:

$$Q(s) = Q'(s) \frac{0.5\tau_n s + 1}{-0.5\tau_n s + 1}. \quad (25)$$

From Eq. (25), we find that $Q(s)$ will include a RHP pole as $s = \frac{2}{\tau_n}$, which is not allowable. Hence, if there exists a time delay that should be considered, the selection of $P_A(s)$ should include the unstable zero $s = \frac{2}{\tau_n}$ such that the finally solution $Q(s)$ has no RHP poles.

Theorem 4. For the Q filter optimized by the standard H_∞ theory, the following properties hold:

1. The relative order of optimized $Q(s)$ is higher than or equal to that of weighting function $W_2(s)$.
2. The optimized $Q(s) \in \mathcal{RH}_\infty$, and it has all the RHP zeros of $P_n(s)$.
3. The open loop transfer function from U_r to U is stable.

Proof. 1. From the optimization function in Eq. (18) and loop shaping theory, we know that $\|1 - Q'(s)\|_\infty < \|\frac{1}{\gamma W_1(s)}\|_\infty$ and $\|Q'(s)\|_\infty < \|\frac{1}{W_2(s)}\|_\infty$. Then we can obtain the following Eq. as:

$$\lim_{\omega \rightarrow \infty} |W_2(j\omega)Q'(j\omega)| < 1, \quad (26)$$

hence, the relative order of $Q'(s)$ is higher than or equal to that of $W_2(s)$. From the definition of $Q'(s)$, we can conclude that $Q(s)$ has the same order property as $Q'(s)$.

2. From the description of H_∞ theory, we know that the closed-loop system of the virtual H_∞ problem is internally stable, then $Q'(s) = \frac{\tilde{P}(s)\tilde{K}(s)}{1 + \tilde{P}(s)\tilde{K}(s)} \in \mathcal{RH}_\infty$. Since $Q(s)$ has the same poles with $Q'(s)$, we can come to a conclusion that $Q(s) \in \mathcal{RH}_\infty$.

Notice that there is no RHP zero-pole cancelation between the virtual plant and controller, $\tilde{L}(s)$ has all the RHP zeros of $\tilde{P}(s)$. Assume that $\tilde{L}(s) = \frac{N_{L-}(s)N_{L+}(s)}{D_L(s)}$, and $D_L(s)$ is coprime with $N_{L-}(s)N_{L+}(s)$, where $N_{L+}(s) = (-0.5\tau_n s + 1) \cdot \prod_i (-s + \xi_i), \text{Re}(\xi_i) > 0$. Then, $Q(s)$ can be expressed as:

$$Q(s) = \frac{N_{L-}(s)N_{L+}(s)}{D_L(s) + N_{L-}(s)N_{L+}(s)} \frac{0.5\tau_n s + 1}{-0.5\tau_n s + 1}, \quad (27)$$

that is, $Q(s)$ have the RHP zeros of $P_n(s)$, then the optimized $Q(s)$ can eliminate all the RHP poles of $P_n^{-1}(s)$.

3. From the equivalent structure in Fig. 3 and the definition of $Q'(s)$, the open loop transfer function from U_r to U is give as: $G_{UU_r} = \frac{1}{1 - Q'(s)}$. According to Eq. (24), this transfer function can be rewritten as: $G_{UU_r} = 1 + \tilde{P}(s)\tilde{K}(s)$. Notice that the virtual plant and

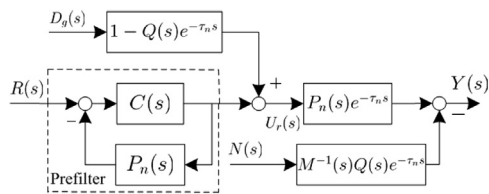


Fig. 3. Equivalent control framework with DOB.

controller are all stable plant, hence, the transfer function G_{UUr} is stable. \square

The above Theorem 3 shows that if the weighting functions and virtual control objective are well-selected, the optimized Q filter satisfies the relative order constraint, and can eliminate the unstable poles of $P_n^{-1}(s)$.

3.4. Further discussions of DOB

The DOB based control structure is proposed to suppress the input external disturbances. However, DOB can also deal with the internal uncertainties. It is very important to analyze how the DOB can compensate the real plant into a nominal one. Here, we regard the nominal plant as

$$Y_n(s) = P_n(s)e^{-\tau_n s}U_r(s), \quad (28)$$

define the error between real plant and nominal one as $e(t) \triangleq y(t) - y_n(t)$, and from (4), we get:

$$E(s) = M^{-1}(s)[(1-Q(s)e^{-\tau_n s})(P(s)e^{-\tau s} - P_n(s)e^{-\tau_n s})U_r(s) - Q(s)P_n^{-1}(s)P(s)e^{-\tau s}N(s) + (1-Q(s)e^{-\tau_n s})P(s)e^{-\tau s}D(s)]. \quad (29)$$

Then, the equivalent control framework is given in Fig. 3, where $D_g(s)$ is the general disturbance caused by both model mismatch and external disturbances:

$$D_g(s) = M^{-1}(s)[(P(s)P_n^{-1}(s)e^{-(\tau-\tau_n)s} - 1)U_r(s) - P(s)P_n^{-1}(s)e^{-(\tau-\tau_n)s}D(s)]. \quad (30)$$

From the transfer function of Eq. (29), we know that the measurement noise can be suppressed if Q filter has low-pass property. If there is no plant uncertainties, the system input of DOB structure u_r will not affect the output y . The question we concern about is how the DOB suppress the disturbance caused by internal uncertainties and external disturbances. Then, we analyze the performance of the proposed DOB under different conditions. In practical applications, a time-varying signal can be represented into sine signals under fourier decomposition. Without loss of generality, we mainly analyze the condition that final-value of d exists, or d is a sine signal.

3.4.1. The final-value of u_r and d exists

The Final-value Theorem can be applied to investigate the output error as:

$$e(\infty) = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} (1-Q(s)e^{-\tau_n s}) \cdot \left[\lim_{s \rightarrow 0} \frac{P_n(s)(P(s)e^{-\tau s} - P_n(s)e^{-\tau_n s})}{P_n(s) + Q(s)(P(s)e^{-\tau s} - P_n(s)e^{-\tau_n s})} \cdot \lim_{s \rightarrow 0} U_r(s) + \lim_{s \rightarrow 0} \frac{P_n(s)P(s)e^{-\tau s}}{P_n(s) + Q(s)(P(s)e^{-\tau s} - P_n(s)e^{-\tau_n s})} \cdot \lim_{s \rightarrow 0} D(s) \right], \quad (31)$$

where $U_r(s)$ and $D(s)$ denote the Laplace transformation of $u_r(t)$ and $d(t)$ such that:

$$\begin{cases} \lim_{s \rightarrow 0} sU_r(s) = u_r(\infty) = \text{constant} \\ \lim_{s \rightarrow 0} sD(s) = d(\infty) = \text{constant}. \end{cases} \quad (32)$$

If the DC gain of $P(s)$ and $P_n(s)$ are both constant, then, we can come to a conclusion that $e(\infty) = 0$ if DC gain of $Q(s)$ is 1. If $P(s)$ or $P_n(s)$ has zeros on imaginary axis, $1-Q(s)e^{-\tau_n s}$ should possess corresponding zeros to hold $e(\infty) = 0$. This can be achieved easily by setting the correspondingly poles in weighting function $W_1(s)$.

3.4.2. External disturbance d is sine signal

At present, the final-value of $e(\infty)$ no longer exists since the transfer function has two poles on imaginary axis. Hence, we hope $1-Q(s)e^{-\tau_n s}$ can eliminate the poles on imaginary axis. This can be acquired if we add the corresponding poles into weighting function $W_1(s)$. Then, the proposed DOB can suppress the output error caused by periodic u_r or d . However, the existence of time delay may have a terrible effect on zeros of $1-Q(s)e^{-\tau_n s}$. The closer the zeros of $1-Q(s)e^{-\tau_n s}$ to poles on imaginary axis, the better disturbance rejection performance against periodic signals will be achieved.

4. Design example

In this section, we present the DOB and controller design procedure specifically. The simulation results are shown to prove the effectiveness of the proposed method.

The NMP process usually exists in many industrial and chemical applications. The dead-time property and unknown disturbances are very hard to deal with. Here, we choose the following NMP process with RHP zeros, dead-time and disturbance to validate the proposed method in this paper. The transfer function of the plant is expressed as:

$$P(s) = \frac{-s+1}{(3s+1)(2s+1)} \cdot \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}, \quad (33)$$

where $\omega_n = 20$ rad, $\xi = 0.25$. The nominal model is given as:

$$P_n(s) = \frac{-s+1}{(3s+1)(2s+1)}. \quad (34)$$

The dead-time and its nominal values are given as: $\tau = 1.1s$, $\tau_n = 1s$. The upper bound of time delay is defined as $0.1s$, which is used to select the weighting function $W_2(s)$. The disturbance acting on the plant is sine signal with the amplitude 1 and frequency 1 rad/s. And then, we give the following procedures of DOB configuration.

Step 1. Select the weighting functions $W_1(s)$ and $W_2(s)$ that reflect the design specifications such as disturbance rejection performance, robust stability and mixed sensitivity design requirements.

We hope the selection of weighting function $W_1(s)$ can suppress the periodic component of external disturbances. Meanwhile, from the description of general disturbance in Eq. (30), $Q(s)$ should possess a pole at the origin to eliminate system uncertainties. From the specified form of external disturbances, we select the weighting function as:

$$W_1(s) = \frac{0.5}{s^2+1} + \frac{1}{s} = \frac{s^2+0.5s+1}{s^3+s}. \quad (35)$$

If there is no prior knowledge of the disturbances, we can simply choose $W_1(s) = \frac{1}{s}$ to attenuate load disturbances.

From the definition of $W_2(s)$, we know that it must be higher than $W_\eta(s)$ and $W_\Delta(s)$ at all frequency range to guarantee robust stability and measurement noise suppression performance. In this simulation, the weighting function $W_2(s)$ should have suppression of -50 dB against measurement noise at frequency of 1000 rad/s with the premise of robust stability. The weighting function is chosen as:

$$W_2(s) = \frac{s+1.5}{2.2}, \quad (36)$$

and from Fig. 4, we notice that the selected $W_2(s)$ satisfies the above requirements.

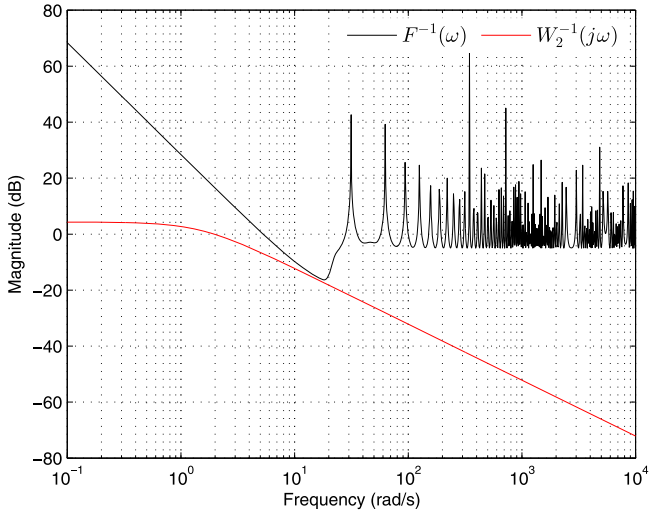


Fig. 4. Selection of weighting function $W_2(s)$.

Step 2. Select a virtually controlled objective $\tilde{P}(s) \in \Sigma_p$ that reflects the relative order with the RHP parts of $P_n(s)$.

From Eq. (21), we choose the virtually controlled objective as:

$$\tilde{P}(s) = \frac{1}{s+2} \cdot \frac{-s+1}{s+1} \cdot \frac{-s+2}{s+2}. \quad (37)$$

Step 3. According to the framework of standard H_∞ problem, the optimal solution $\tilde{K}_{opt}(s)$ is obtained due to the proposed optimization problem in Eq. (20).

Under the standard H_∞ theory, we get the virtually optimal controller while $\gamma = 0.1804$ as:

$$\tilde{K}(s) = \frac{42.1672(s+3)(s+2)(s+1)}{s(s+21.99)(s^2+1)} \cdot \frac{s^2-0.3595s+0.9505}{s^2+2.269s+14.78}. \quad (38)$$

Here, notice that $\tilde{K}(s)$ is proper and can eliminate all the poles of $\tilde{P}(s)$.

Step 4. Obtain optimal loop function $\tilde{L}(s)$ and then compute the optimal Q filter by Eqs. (24) and (25).

From Step 3, we obtain Q filter as:

$$Q(s) = \frac{-42.1672(s+2)(s-1)}{(s+19.16)(s+2.293)(s^2+2.41s+1.877)} \cdot \frac{s^2-0.3595s+0.9505}{s^2+0.3922s+0.9724}. \quad (39)$$

Here, we find that the Q filter can eliminate the RHP poles of $P_n^{-1}(s)$, which meets Requirement 2 of Theorem 1.

Step 5. Design a controller $C(s)$ as the prefilter based on the nominal plant $P_n(s)$. Requirement 1 of Theorem 1 should be satisfied.

The controller is designed based on H_2 theory as:

$$C(s) = \frac{6s^2+5s+1}{0.2s^2+4.2s}. \quad (40)$$

Then we get the prefilter as:

$$\frac{C(s)}{1+C(s)P_n(s)} = \frac{6s^2+5s+1}{0.2s^2+3.2s+1}, \quad (41)$$

which satisfies Requirement 1 of Theorem 1. The whole control structure based on DOB has been successfully constructed.

If we do not consider the time delay in Q filter design procedure, the following Q filter $Q'(s)$ will be obtained:

$$Q'(s) = \frac{-2.2012(s-1)(s^2-0.2707s+0.7714)}{(s^2+2.732s+2.027)(s^2+0.3414s+0.8379)}. \quad (42)$$

We also use the method proposed in [24] to design the Q filter. The following two Q filters with same relative order as $Q(s)$ are

given as:

$$\begin{cases} Q_{b1}(s) = \frac{1}{(\tau_1 s)^2 + 2\tau_1 s + 1} \cdot \frac{-s+1}{s+1}, \\ Q_{b2}(s) = \frac{3\tau_2 s + 1}{(\tau_2 s)^3 + 3(\tau_2 s)^2 + 3\tau_2 s + 1} \cdot \frac{-s+1}{s+1}. \end{cases} \quad (43)$$

where $\tau_1 = 0.25$, $\tau_2 = 0.85$ is selected to guarantee the robust stability.

The comparison of frequency magnitude of the above Q filters are expressed in Figs. 5 and 6. From the frequency responses in Fig. 5, it is verified that designed Q filters $Q(s)$ and $Q'(s)$ under weighting function $W_2(s)$ satisfies the robust stability condition. Meanwhile, the time constants in $Q_{b1}(s)$ and $Q_{b2}(s)$ are well-selected for robust stability condition. Since time delay is an allpass portion, designed $Q'(s)$ without consideration of time delay will not affect the robust stability of the system. Then we analyze the disturbance suppression performance. Nevertheless, if we do not consider the time delay during Q filter design procedure, the time delay will bring frequency response of $1-Q(s)e^{-\tau_n s}$ with a translation. This is revealed in Fig. 6. From the enlarged frequency response of $1-Q(s)e^{-\tau_n s}$, only the Q filter which considers the time delay will lead the frequency response of $1-Q(s)e^{-\tau_n s}$ a sharp drop at frequency of 1 rad/s, with its magnitude almost -50 dB at this frequency. According to Eq. (4), we know that this Q filter can suppress the constant disturbances as well as periodic disturbances with frequency of 1 rad/s. Nevertheless, the magnitude of other three filters is larger at frequency of 1 rad/s in Fig. 6, which means $1-Q'(s)e^{-\tau_n s}$ cannot eliminate the poles of $D(s)$ on imaginary axis. This indicates that the disturbance rejection performance of $Q(s)$ is the best of these Q filters. Especially, we notice that the frequency response of $Q'(s)$, $Q_{b1}(s)$ and $Q_{b2}(s)$ at $\omega = 1$ rad are all larger than 0 dB, which means these three filters will even enlarge the influence caused by external disturbances.

In the simulations, external disturbances with constant and periodic value are taken into account, respectively. Fig. 7(a) shows the step response of the system with constantly external disturbances, whereas the performance with periodic disturbance is presented in Fig. 7(b). Since the DC gain of the mentioned Q filters is 1, these Q filters can eliminate the external disturbances with constant value easily. We also find that DOB with $Q(s)$ has better performance of periodic disturbance attenuation. Since the frequency response of $Q'(s)$, $Q_{b1}(s)$ and $Q_{b2}(s)$ at $\omega = 1$ rad are all larger than 0 dB, in other words, these three filters will enlarge the output error caused by periodic external disturbances. Hence, to deal with the periodic external disturbances, a well-designed Q filter is especially important for NMP systems. The estimation

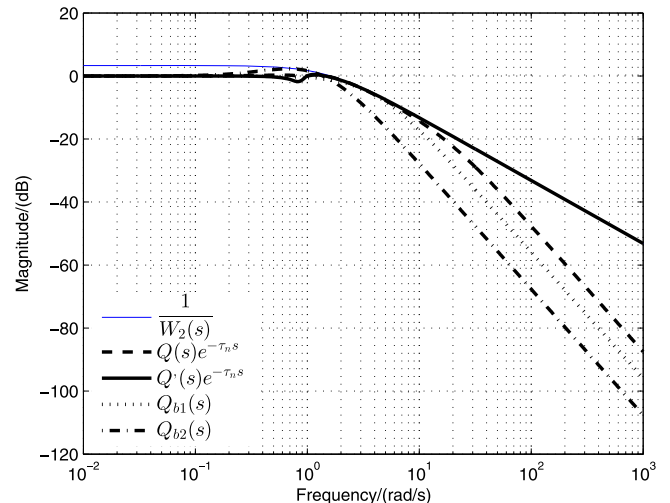


Fig. 5. Robust stability against system uncertainties.

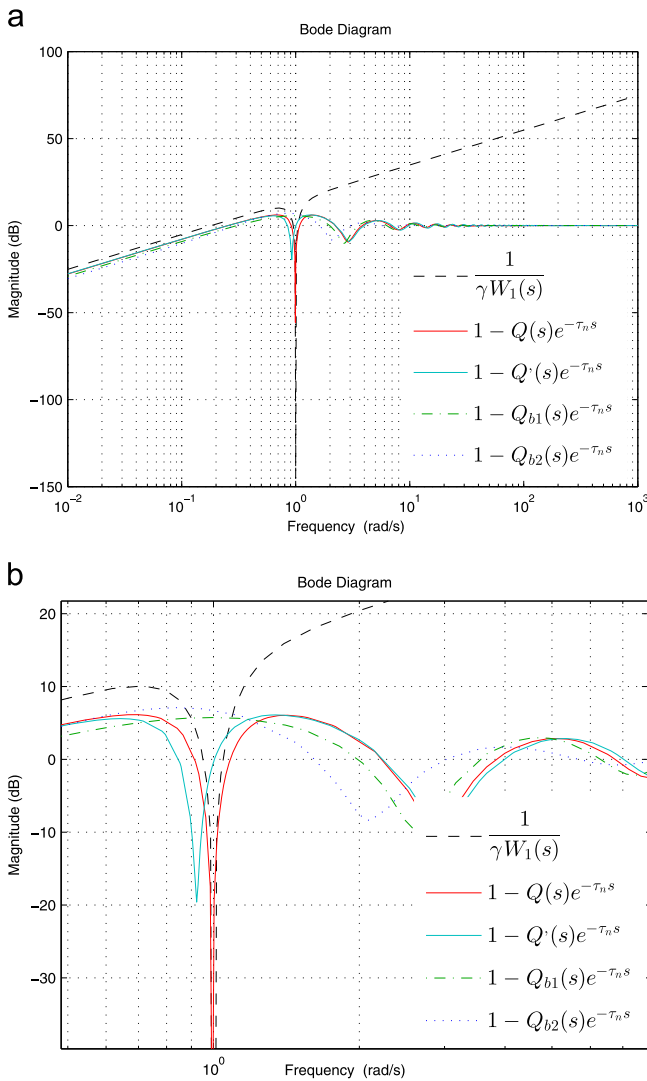


Fig. 6. Comparison of disturbance rejection performance. (a) Frequency magnitude responses. (b) Enlarged view.

effect with periodic disturbance is shown in Fig. 8, it is clearly that the proposed DOB can estimate the general disturbance D_g accurately within 10 second. The control output in shown in Fig. 9, with the estimation of DOB, the proposed controller can adjust the control output according to the disturbances to increase the control accuracy. Fig. 7(b) also shows that the control accuracy of the overall system increase with the convergence of DOB.

5. Application of mechanical system

In this section, a rotary mechanical system is introduced to verify the effectiveness of the proposed control scheme. The rotary mechanical system consists of inertias, dampers, torsional springs, a timing belt, pulleys, and gears. The transfer function of the plant is shown as follows [27]

$$P(s) = \frac{123.853 \times 10^4 (-s + 3.5)}{(s^2 + 6.5s + 42.25)(s + 45)(s + 190)} \quad (44)$$

Notice that the poles $s = -45$ and $s = -190$ are far away from the dominant conjugate poles, and consider the uncertainties of

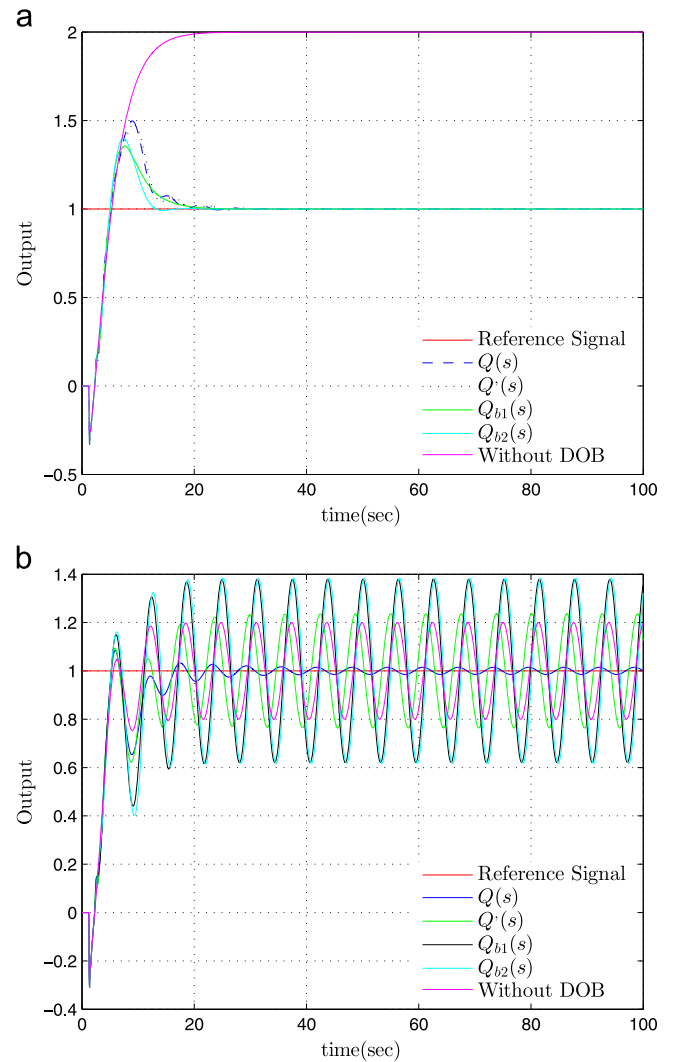


Fig. 7. Step response with external disturbances. (a) Constantly disturbance. (b) Periodic disturbance.

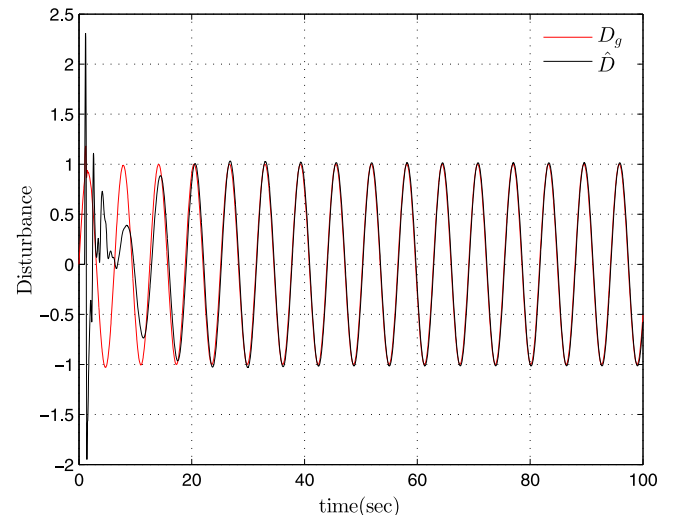


Fig. 8. Estimation effect of periodic disturbance. the model. The nominal plant is selected as

$$P_n(s) = \frac{144.86(-s + 3)}{(s^2 + 6.5s + 42.25)} \quad (45)$$

To suppress the constantly external disturbances under the condition of guarantying the robust stability, the weighting functions are given as

$$\begin{cases} W_1(s) = \frac{1}{s} \\ W_2(s) = \frac{0.2s+3}{9} \end{cases} \quad (46)$$

Since the relative order of $P_n(s)$ is 1, the relative order of $W_2(s)$ is chosen as 1 to make the relative order of $Q(s)$ same as that of $P_n(s)$. Then, follow the design specifications in Section 4, by introduce the virtually control objective

$$\tilde{P}(s) = \frac{1}{s+2} \cdot \frac{-s+3}{s+3}, \quad (47)$$

the virtual controller is optimized as

$$\tilde{K}(s) = \frac{45(s+3)(s+2)}{s(s+64.57)}, \quad (48)$$

and the Q filter is expressed as

$$Q(s) = \frac{45(-s+3)}{s^2+19.57s+135}. \quad (49)$$

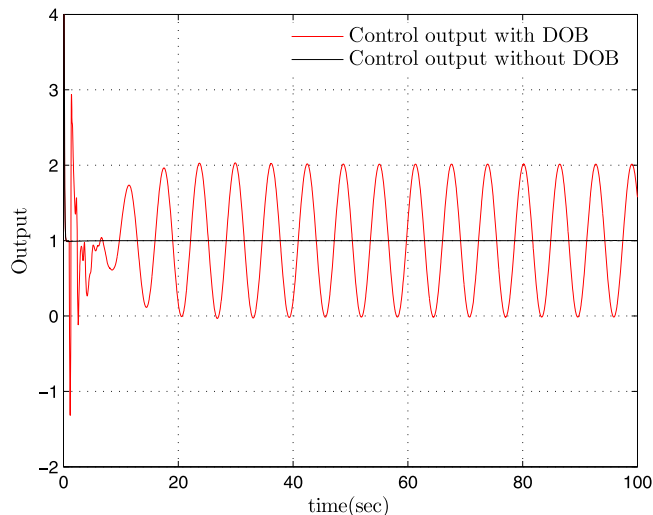


Fig. 9. Control output with periodic disturbance.

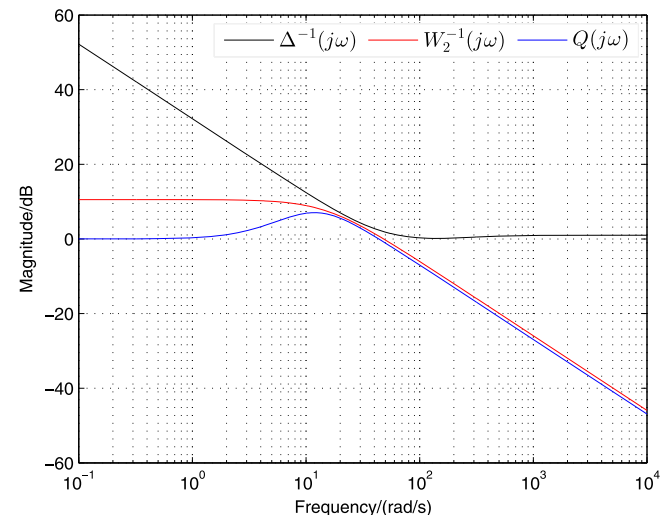


Fig. 10. Verification of robust stability condition.

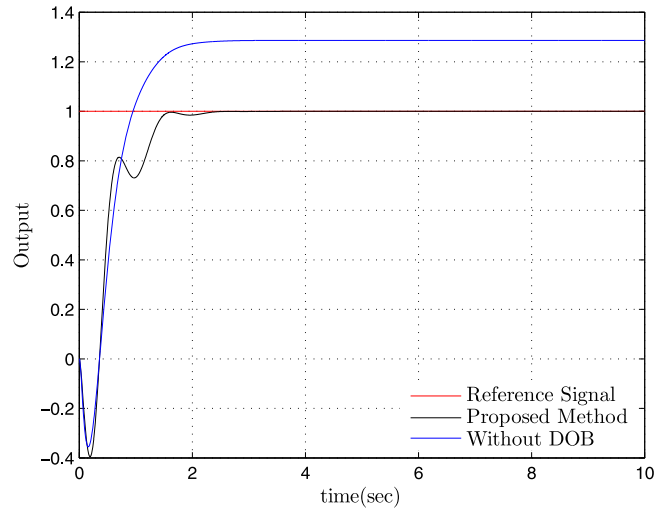


Fig. 11. Step response with constantly disturbances.

Then, the controller is designed based on H_2 theory as

$$C(s) = \frac{s^2+6.5s+42.25}{144.86(0.2s^2+2.6s)}. \quad (50)$$

Finally, we get the prefilter as

$$\frac{C(s)}{1+C(s)P_n(s)} = \frac{s^2+6.5s+42.25}{144.86(0.2s^2+1.6s+3)}. \quad (51)$$

Fig. 10 shows the robust stability condition of the closed-loop system with DOB. It is verified from Fig. 10 that the weighting function that reflects the robust stability condition $W_2(s)$ is well selected. It is also shown that the optimized Q filter satisfies the robust stability condition very well. The output in Fig. 11 shows the control performance of control system in the presence of constantly disturbance. Without DOB, there exists steady-state error caused by external disturbances and system uncertainties. The designed DOB can eliminate the steady-state error successfully.

We can easily calculate the DC gains of the real and nominal plants with $s=j0$ as: $P(j0) = 12.0$, $P_n(j0) = 10.3$. It is clear that the DC gain of the nominal model does not match that of the real system. However, due to the simulation result shown in Fig. 10, the proposed method can enable the output to converge to the reference signal successfully without steady-state error. This design example shows that the mismatch of the DC gains of the real and nominal plant will not bring the control system with uncorrected error between the output and the reference signal.

6. Conclusions

In this paper, we mainly focus on the DOB based controller designed for stable NMP systems with both RHP zeros and time delay. A systematic DOB design strategy is proposed. We synthesize the internal and robust stability constraints, relative order and mixed sensitivity design requirements together to establish an optimization function. The standard H_∞ theory is applied to obtain the optimal solution. At last, we consider a NMP process and present the design procedure specifically to show that all the constraints can be guaranteed based on the design procedures. Simulations are carried out to show that the proposed method can be successfully employed for stable NMP systems. It is verified that the proposed method can be used effectively under different conditions of external disturbances.

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