Robust Disturbance Observer for Two-Inertia System

Jong Nam Yun, Jianbo Su, *Senior Member, IEEE*, Yong Il Kim, and Yong Chun Kim

***Abstract*—This paper proposes a method for designing disturbance observer (DOB) in vibration suppression control of two-inertia system. The key to a vibration suppression control of two-inertia system is eliminating the effect of vibration modes of the plant. For this purpose, feedback controller is designed to cancel the stable vibration modes of plant. However, this cancellation fails due to model error from parameter variations. In this paper, the DOB is employed to compensate for the effect of parameter variations. The Q-filter of DOB is designed with the following specifications: optimal disturbance suppression performance with restricted cutoff frequency; robust stability of closed-loop system against modeling error between the plant and its nominal model; structural restriction of Q-filter itself such as relative order and internal model order. This paper proposes a systematic and straightforward method to attain the above design specifications using standard H_∞ control framework. Experimental results verify that the proposed DOB can improve the transient response and effectively suppress the vibration and improve external disturbance removing performance as well.**

***Index Terms*—Disturbance observer (DOB), high-performance robust motion control, standard H_∞ control problem, two-inertia system, vibration suppression.**

I. Introduction

VIBRATION suppression in most mechanical systems is an important theme for improving control effectiveness. In particular, for robot joints with flexibility and torsion shaft which are modeled as spring, it is critical for control algorithms to achieve vibration suppression performance. However, with traditional control strategies, it is hard to attain good vibration suppression performance and fast and smooth transient response simultaneously for these plants. In order to testify the performance of control algorithm, the two-inertia system is often used as a benchmark plant [1], [2].

In motion control system, the disturbance observer (DOB)-based controller is widely used to estimate and suppress exotic disturbance and the equivalent disturbance due to parameter variations [3], [4]. It also is used to decentralize multi-input multi-output (MIMO) coupled dynamics of robot manipulator into individual decoupled servo systems [5]. DOB is able to improve disturbance rejection performance and command following characteristics by means of high gain of nearly infinity at low frequencies. DOB consists of a nominal model and a low-pass filter, called Q-filter, thus selection of the nominal model and design of the Q-filter are key issues of DOB configuration. Q-filter has some structural properties: order, relative order, internal model order [6], and some frequency properties: cutoff frequency (bandwidth) and peak magnitude around this frequency. Because of simple structure and possibility of intuitive tuning of loop gains, its more common practices have been found in many high performance control systems [7]–[11]. In order to improve response and accuracy of multi-inertia system, there are many contributions to the study of vibration suppression such as traditional feedback control [12], [13]. DOB-based control techniques have also shown outstanding control effectiveness in this field [14]–[17]. Resonance ratio control by DOB with traditional Q-filter [15], MIMO DOB design method on multi-degrees of freedom (DOF) system [16] and fractional-order DOB [17] are the results of study and application of DOB to two-inertia system. In the area of vibration suppression control, independent modal space control method is also widely used. In the scheme, the resonance frequency and damping ratio of vibration modes is modified to desired ones by input force compensation [18], [19].

In most of the studies on DOB, only robust stability of DOB local loop to parameter variations was considered, but that of closed loop has not been considered [17], [20]. DOB design method based on H_∞ norm optimization can strictly guarantee the robust stability, but its solving procedure is based on numerical computation because of structural restrictions of DOB itself [21], [22]. Extended H_∞ control design method for DOB type control system was developed in [23], where DOB is regarded as an alternative design of integral controller. For design of the controller, a systematic method for solving the servo problem with weighting functions including imaginary axis poles is presented. On the other hand, the systematic and direct procedure of designing DOB as an inner loop of 2 DOF system was proposed in [24], but only robust stability of DOB local loop was discussed.

This paper proposes a systematic design method of robust DOB for suppressing vibration in two-inertia system using H_∞ design technique. In two-inertia system, to get nonvibration response, the vibration modes of plant should be canceled by controller. However, if parameter variations occur, this cancellation will be failed. It can be resolved by the ability of DOB to fix the perturbed plant to nominal model. To attain this purpose, DOB should not only guarantee closed robust stability but also have strong performance of suppressing equivalent disturbance

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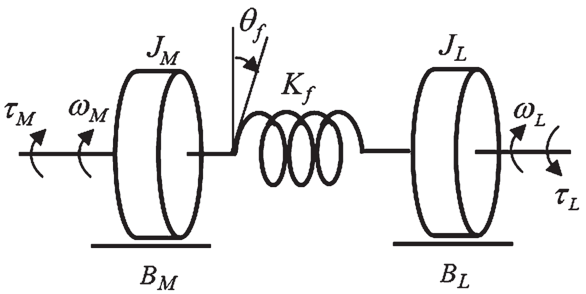

Fig. 1. Model of two-inertia system.

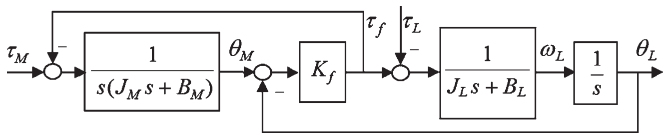

Fig. 2. Block diagram of the two inertia system.

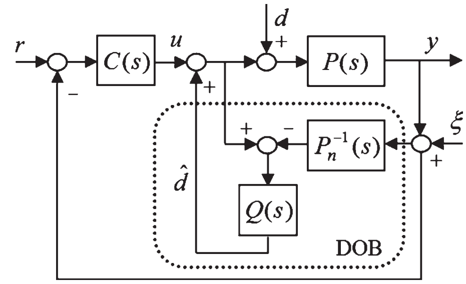

Fig. 3. Feedback control system with DOB.

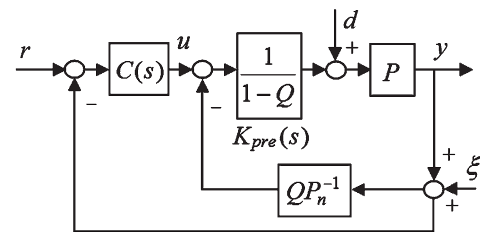

Fig. 4. Equivalent system of DOB-based control system.

related to parameter variations. Considering these points, DOB for control of two-inertia system should be carefully designed to attain the following performances: 1) Q-filter of DOB should satisfy robust stability of closed loop to parameter variations; 2) Q-filter should also have maximal disturbance suppression performance, under the robust stability condition; 3) Q-filter itself should satisfy the structural restrictions such as order, relative order and internal model order. In order to get systematic and straightforward solution of optimal Q-filter realizing these purposes, a new robust DOB design method is proposed using standard H_∞ control design framework.

The remainder of this paper is organized as follows: in Section II, mechanical model of two-inertia system and overview of control strategy for this system are discussed. Section III provides a systematic design method of Q-filter satisfying design specifications on the system in the standard H_∞ design framework. Section IV gives results of design and experiment on the torsional shaft experimental equipment, followed by conclusions in Section V.

II. Mechanical Model of Two-Inertia System and DOB-Based Control System

A. Mechanical Model of Two-Inertia System

Mechanical model of two-inertia system is shown in Fig. 1 and its block diagram in Fig. 2. It has five parameters: two inertia moments J_M and J_L of fly wheels of driving motor side and load side, two viscous friction coefficients B_M and B_L of the two wheels and elasticity coefficient K_f of torsional shaft. Variables τ_M and θ_M are torque of driving motor and angle of driving shaft, respectively; τ_L and θ_L are load torque and angle of load shaft; $\theta_f = \theta_M - \theta_L$ is torsion angle. In this paper, positioning control problem in the presence of external disturbances is considered. Equations of this system is as follows:

$$\begin{cases} J_M\ddot{\theta}_M + B_M\dot{\theta}_M = \tau_M - \tau_f \\ J_L\ddot{\theta}_L + B_L\dot{\theta}_L = \tau_f - \tau_L = K_f(\theta_M - \theta_L) - \tau_L. \end{cases} \quad (1)$$

Two situations will be considered, i.e., the sensor for feedback is equipped in load shaft and in driving shaft. Then, the transfer function $G_{LM}(s)$ from driving torque to rotation angle of load shaft and $G_{MM}(s)$ from driving torque to one of the driving motor shaft are

$$G_{LM}(s) = \frac{1}{s} \cdot \frac{K_f}{a_{G3}s^3 + a_{G2}s^2 + a_{G1}s + a_{G0}} \quad (2)$$

$$G_{MM}(s) = \frac{1}{s} \cdot \frac{b_{G2}s^2 + b_{G1}s + b_{G0}}{a_{G3}s^3 + a_{G2}s^2 + a_{G1}s + a_{G0}} \quad (3)$$

where $a_{G3} = J_M J_L$, $a_{G2} = J_M B_L + J_L B_M$, $a_{G1} = K_f(J_M + J_L) + B_M B_L$, $a_{G0} = K_f(B_M + B_L)$, $b_{G2} = J_L$, $b_{G1} = B_L$ and $b_{G0} = K_f$, respectively.

From (2) and (3), it is clear that resonance frequency and damping rate of the system are directly related to the parameters such as J_L and K_f, etc. The less the viscous coefficients are, the stronger the system's vibratility is. The cancellation of the plant's oscillation modes by the controller can remove the vibration of dynamic response. However, parameter variations may lead to failure of this cancellation of the oscillation modes, thus resulting in appearance of vibration.

B. Control System for Two-Inertia System

DOB can fix the extended plant to nominal model by means of high feedback gain of nearly infinity at low frequencies. Using this property of DOB, the cancellation of resonance modes of plant by controller can be maintained in spite of parameter variations. Fig. 3 shows DOB-based control system, and its equivalent system is shown in Fig. 4, where $C(s)$, $P(s)$, $P_n(s)$, and $Q(s)$ are feedback controller, plant, nominal model, and Q-filter, respectively. $K_{pre}(s) = (1 - Q(s))^{-1}$ is the equivalent controller. From the block diagrams in these figures, it is verified that

$$y = \left[1 - Q + PQP_n^{-1}\right]^{-1}\left[Pu + (1 - Q)Pd\right]. \quad (4)$$

It means that if $Q(s) \simeq 1$ (at low frequencies), the extended transfer function from u to y is identical with $P_n(s)$ and the function from d to y equals zero no matter what $P(s)$ is.

To build control loop of two-inertia system by DOB, firstly, nominal plant $P_n(s)$ should be selected, then feedback controller $C(s)$ should be designed. Selections of $P_n(s)$ and $C(s)$ are mutually dependent. Since $P_n(s)$ should be chosen near to the real plant $P(s)$ and preserve its order [5], $P_n(s)$ includes the resonance modes and inverse resonance modes of $P(s)$

$$P_n(s) = \frac{N_{n1}(s)N_{n2}(s)}{M_{n1}(s)M_{n2}(s)} \tag{5}$$

where $M_{n2}(s)$ and $N_{n2}(s)$ are stable resonance polynomial and stable inverse resonance polynomial which should be canceled by controller $C(s)$. In two-inertia system, $N_{n1}(s)$ is constant and $M_{n1}(s) = s$ on the condition that the numerator and denominator polynomials of second factors in (2)and (3) are stable. $C(s)$ may consist of the polynomials to cancel the stable polynomials of the resonance modes without unstable zero-pole cancellation, i.e.,

$$C(s) = \frac{N_{C1}(s)M_{n2}(s)}{M_{C1}(s)N_{n2}(s)}. \tag{6}$$

Thus, the closed-loop function $G_{yr}(s)$ from r to y is

$$G_{yr}(s) = \frac{L(s)}{1 + L(s)} = \frac{N_{n1}N_{C1}}{N_{n1}N_{C1} + M_{n1}M_{C1}} \tag{7}$$

where $L(s) = P_n(s)C(s)$ is open function in nominal state. In DOB-based 2 DOF control system design, outer loop controller's design does not take feedback properties such as disturbance suppression and low sensitivity into account, because they are considered in DOB inner loop design. Thus, the design of $C(s)$ may consider only command tracking performance, and we can use the polynomial $M_{C1}(s)$ and $N_{C1}(s)$ to make $G_{yr}(s)$ equal to a reference model $G_M(s)$ which has desirable response. $G_M(s)$'s time constant should be selected to satisfy original robust stability without DOB, as explained later.

This approach, however, is possible only when plant and nominal model are identical. If plant is away from nominal model due to parameter variations, the controller fails to cancel resonance modes of the plant. DOB can be used to maintain the extended plant match the nominal model. To attain this, DOB should have maximal disturbance suppressing performance under the robust stability condition.

III. Design of Q-Filter With Robustness Specifications

In the second stage of designing DOB-based controller for two-inertia system, the Q-filter should be designed. In fact, the design of Q-filter is a key to design of DOB-based control system, because it determines both disturbance rejecting performance and robustness to parameter variations. In design of Q-filter, the following performances should be taken into account.

i) It must satisfy robust stability of closed-loop system to parameter variations. It is, in particular, important for two-inertia system with changeable load.

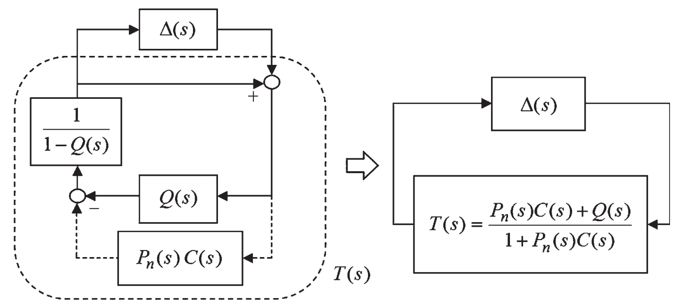

Fig. 5. Robust stability analysis.

ii) It should be designed so that DOB could have optimal disturbance rejecting performance or equivalently minimal sensitivity at low frequencies (control band).

iii) It should satisfy corresponding order restrictions such as order, relative order, and internal model order [6]. Relative order is required from realizability of $Q(s)P_n^{-1}(s)$ in Fig. 4, while internal model order is related with the order of exotic disturbance to be removed.

A. Robust Stability

In design of DOB, selection of Q-filter satisfying stability of closed-loop system is the most important problem. In many cases, first- or second-order system have been employed as Q-filter, in which the time constant is used to set cutoff frequency blocking high band of noise or modeling error [5]. This method is easy to be used in industrial applications. On the other hand, it is recognized that the higher order of Q-filter is, the more possible to improve the disturbance suppressing performance under the condition of the same pass band. However, how to adjust systematically the coefficients of high order Q-filter becomes an academic issue, since Q-filter should satisfy some structural restrictions such as relative order, internal model order, etc. A systematic procedure to resolve this problem has been developed in [24], which only guarantees stability of DOB inner loop. Based on the procedure, a systematic solving method guaranteeing the robust stability of closed-loop system is developed as follows.

Suppose that model uncertainty can be treated as a multiplicative perturbation, that is

$$P(s) = P_n(s)\left[I + \Delta(s)\right] \tag{8}$$

where variation $\Delta(s)$ is assumed to be stable. The DOB inner loop is robust stable if (see solid parts of diagram in Fig. 5)

$$\overline{\sigma}\left(\Delta(j\omega) \cdot Q(j\omega)\right) < 1, \quad \forall\omega \tag{9}$$

where $\overline{\sigma}(\cdot)$ represents the maximum of singular value.

Robust stability design based on DOB inner loop seems to be straightforward with clear physical interpretation, but to meet (9) itself is not the final purpose of stability. DOB loop is only a part of the overall control system. Robust stability to model perturbation should be considered with feedback control loop being connected to DOB loop. From Fig. 5 (including dashed parts), it is clear that the transfer function from the output of

$\Delta(s)$ to its input, i.e., the complementary sensitivity function of closed-loop system is

$$T(s) = \frac{P_n(s)C(s) + Q(s)}{1 + P_n(s)C(s)} = \frac{P_n(s)C(s) + T_{DOB}(s)}{1 + P_n(s)C(s)} \quad (10)$$

where $T_{DOB}(s) = Q(s)$ is the complementary sensitivity function of DOB inner loop. Thus, feedback control loop with DOB is robust stable against perturbation $\Delta(s)$ if the following condition is satisfied:

$$\overline{\sigma}\left(\Delta(j\omega) \cdot T(j\omega)\right) < 1, \qquad \forall\omega. \quad (11)$$

If an upper limit function $W_U(s)$ is chosen so that

$$|\Delta(j\omega)| \le |W_U(j\omega)|, \qquad \forall\omega \quad (12)$$

then satisfaction of the following condition guarantees the robust stability of the closed system:

$$\left\| W_U(s) \frac{P_n(s)C(s) + Q(s)}{1 + P_n(s)C(s)} \right\|_\infty < 1. \quad (13)$$

From (13), it is known that robust stability depends on both feedback controller $C(s)$ and Q-filter $Q(s)$. Therefore, feedback controller $C(s)$ should be designed, at first, so that system could achieve desired command tracking performance as shown in Section II-B, and then $Q(s)$ could be designed so that robust stability and optimal disturbance suppression performance can be obtained.

B. Sensitivity of Feedback Control System

If we assume $P(s) = P_n(s)$ in Fig. 3, the output y of the closed-loop system can be written as

$$y = \frac{P_n(s)C(s)}{1 + P_n(s)C(s)} \cdot r + \frac{P_n(s)(1 - Q(s))}{1 + P_n(s)C(s)} \cdot d + \frac{P_n(s)C(s) + Q(s)}{1 + P_n(s)C(s)} \cdot \xi. \quad (14)$$

The sensitivity function of closed-loop system is defined as

$$S(s) = \frac{1 - Q(s)}{1 + P_n(s)C(s)} = \frac{S_{DOB}(s)}{1 + P_n(s)C(s)} \quad (15)$$

where $S_{DOB}(s) = 1 - Q(s)$ is the sensitivity function of DOB inner loop. From the above equations, it comes that the sensitivity of closed feedback loop to model perturbation and the effect of disturbance d on tracking error depend on sensitivity function (15), while the effect of sensor noise on the output depends on complementary sensitivity function (10). Thus, to reduce the effect of disturbance and sensor noise on the closed loop as small as possible under the condition of guaranteeing robust stability, $Q(s)$ should be designed by

$$\max \gamma, \quad \min_{Q(s)} \left\|\gamma \cdot W_P(s) \cdot S(s)\right\|_\infty < 1 \quad (16)$$

subject to (13), where $W_P(s)$ is a weighting function restricting the sensitivity at low frequencies and $\gamma > 0$ is a positive number. Then, Q-filter design problem for optimal performances and robust stability of closed-loop system can be defined as

$$\max \gamma, \quad \min_{\substack{Q(s) \in \Omega_{n,k,q} \\ Q(s) \in RH_\infty}} \left\| \begin{bmatrix} \gamma \cdot W_p(s) \frac{1 - Q(s)}{1 + P_n(s)C(s)} \\ W_U(s) \frac{P_n(s)C(s) + Q(s)}{1 + P_n(s)C(s)} \end{bmatrix} \right\|_\infty < 1 \quad (17)$$

where

$$\Omega_{n,k,q} = \left\{ F(s) | F(s) = \frac{\sum_{j=0}^{m} b_j s^j}{\sum_{i=0}^{n} a_i s^i}, \; a_n > 0, \; b_m > 0, \; k = n - m, \quad a_l = b_l \quad (l = 0, 1, \ldots, q) \right\} \quad (18)$$

is the set of transfer functions that satisfies the restrictions of order n, relative order k and internal model order q (system can compensate time quantic disturbance of order q).

C. Transformation to Standard H_∞ DOB Design Problem

Q-filter design problem (17) with robustness specifications for closed loop is difficult to solve in a systematic and straightforward way since it has not structure satisfying precondition of standard H_∞ control problem. Such a problem could only be solved by numerical computation solving algorithm [21], [22]. Transformation of (17) to standard H_∞ control problem can provide systematic solving procedure and guarantee globally optimal solutions. Robust stability condition (13) can be rewritten as

$$|L(j\omega) + Q(j\omega)| < \left|W_U^{-1}(j\omega)\left(1 + L(j\omega)\right)\right|, \qquad \forall\omega \quad (19)$$

where $L(s)$ is the same as the open-loop transfer function of (7). By the property of absolute value of complex number

$$|L(j\omega) + Q(j\omega)| < |L(j\omega)| + |Q(j\omega)|, \qquad \forall\omega. \quad (20)$$

Thus, (19) always holds if $Q(s)$ satisfies the following inequality:

$$|Q(j\omega)| < \left|W_U^{-1}(j\omega)\left(1 + L(j\omega)\right)\right| - |L(j\omega)|, \qquad \forall\omega. \quad (21)$$

In general, robust stability definition by partitioning of absolute value of complex summation such as (20) may be somehow conservative. However, in most feedback control systems, since the phase delays of $Q(s)$ and $L(s)$ are nearly similar around cutoff frequency of $Q(s)$ and the frequencies more than that, difference between two sides of (20) are small, thereby, conservative of robust stability sufficient condition (21) is also slight. Select a scalar weighting function $W_Q(s)$ such that

$$\left|W_Q^{-1}(j\omega)\right| \le \left|W_U^{-1}(j\omega)\left(1 + L(j\omega)\right)\right| - |L(j\omega)| \triangleq E(\omega), \qquad \forall\omega. \quad (22)$$

Function $E(\omega)$ is determined by upper limit function $W_U(s)$ of model uncertainty and nominal open-loop function $L(s)$. It means that given model uncertainty bound, we can obtain

$E(\omega)$ after designing feedback loop for attaining tracking performance. It is clear that if the closed-loop system satisfies the general robust stability sufficient condition (without DOB)

$$\|W_U(s)T_0(s)\|_\infty < 1$$

or equivalently

$$\left|\frac{L(j\omega)}{1+L(j\omega)}\right| < \left|W_U^{-1}(j\omega)\right|, \qquad \forall\omega \tag{23}$$

then $E(\omega) > 0,\ \forall\omega$, where $T_0(s) = L(s)/(1+L(s)) \in RH_\infty$ (stable and proper) is complementary sensitivity function of closed loop without DOB. This condition could be attained by suitable design of feedback controller $C(s)$, e.g., adjusting the time constant of $G_M(s)$ to be matched with $G_{yr}(s)$. By (21) and (22), the closed-loop system is robustly stable if

$$\|W_Q(s)T_{DOB}(s)\|_\infty = \|W_Q(s)Q(s)\|_\infty < 1. \tag{24}$$

On the other hand, (16) can be rewritten as

$$\left|\gamma\frac{W_P(j\omega)}{1+L(j\omega)}(1-Q(j\omega))\right| < 1, \qquad \forall\omega \tag{25}$$

which can be regarded as a constraint on $S_{DOB}(s)$. If we select a weighting function $W_C(s)$ such that

$$\left|\frac{W_P(s)}{1+L(j\omega)}\right| \le |W_C(j\omega)|, \qquad \forall\omega \tag{26}$$

then (27) can be a sufficient condition for (16)

$$\max\gamma, \quad \min_{Q(s)} \|\gamma\cdot W_C(s)\cdot S_{DOB}(s)\|_\infty < 1. \tag{27}$$

Now, we can replace the robust DOB design problem (17) for optimal performances of closed-loop system by

$$\max\gamma, \min_{\substack{Q(s)\in\Omega_{n,k,q}\\ Q(s)\in RH_\infty}} \left\|\begin{bmatrix}\gamma\cdot W_C(s)\,(1-Q(s))\\ W_Q(s)Q(s)\end{bmatrix}\right\|_\infty < 1. \tag{28}$$

The optimal problem (28) is in the form of standard H_∞ DOB design problem in [24]. It means that original design problem (17) of Q-filter considering closed-loop system can be transformed into standard design problem of DOB inner loop itself. Thus, we can solve it by systematic and straightforward solving procedure of standard H_∞ control framework such as DGKF method [25]. In particular, we can easily realize structure restrictions (18) of Q-filter by this procedure.

IV. Experiment

The configuration of the experimental system is shown in Fig. 6, where two fly wheels are fixed to two ends of a torsional shaft and driving motor and load motor are mounted in driving side and load side separately. Table I shows the values of the each parameters of the experimental system. Two sensors are equipped on driving motor shaft and load motor shaft: the first sensor (encoder 1) is to detect rotation angle of load shaft, while the second sensor (encoder 2) is to detect that of motor axis. Two situations which use each other sensor for feedback control are considered below. In the design process, robust stability and sensitivity to variations of J_L and K_f are considered, as well as the performance of suppressing exotic disturbances (generated by load motor).

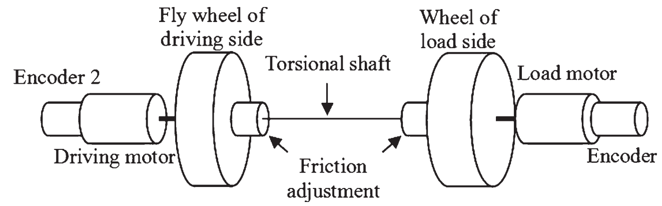

Fig. 6. Configuration of experiment system.

TABLE I
Parameters of the Two Inertia System

J_M	2.267×10^{-3}	$Kg\cdot m^2$
B_M	0.021	$N\cdot m\cdot s/rad$
J_L	$3.5\sim7.0\times10^{-3}$	$Kg\cdot m^2$
B_L	0.019	$N\cdot m\cdot s/rad$
K_f	$62\sim82$	$N\cdot m/rad$

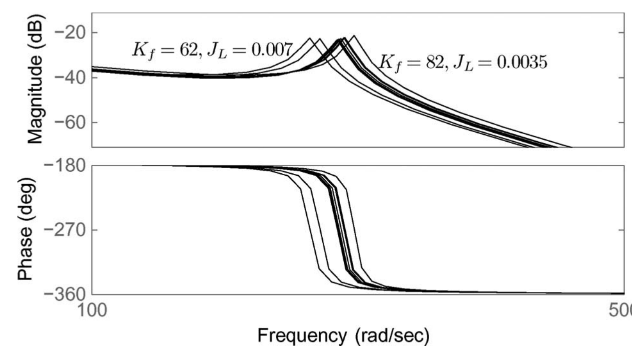

Fig. 7. Frequency responses of $G_{LM}(s)$ for $J_L = 0.0035 \sim 0.007$ and $K_f = 62 \sim 82$.

A. Case of Using Sensor of Load Side

In design of control using load side sensor, $G_{LM}(s)$ of (2) is used as the model of plant for robust stability analysis in Section III. Fig. 7 shows frequency responses of $G_{LM}(s)$ for variations of parameters: $J_L = 0.0035 \sim 0.007$, $K_f = 62 \sim 82$. It shows that the resonance modes sensitively depend on the parameter variations.

1) Nominal Model and Feedback Controller: Considering $M_{n1} = s$, $N_{n2} = 1$, $N_{n1} = K_{fn}$, and $M_{n2} = a_{G3}s^3 + \ldots + a_{G0}$, reference model $G_M(s)$ for $G_{yr}(s)$ can be selected as the following fourth-order modified binomial model:

$$G_M(s) = \frac{1}{(\sigma_M s)^4 + c_1(\sigma_M s)^3 + c_2(\sigma_M s)^2 + c_3(\sigma_M s) + 1} \tag{29}$$

where $c_1 = 2.4$, $c_2 = 6$, $c_3 = 4.8$ and time constant $\sigma_M = 0.02$ is selected by considering general robust stability (without DOB). It has step response of nonvibration, nonovershoot, and

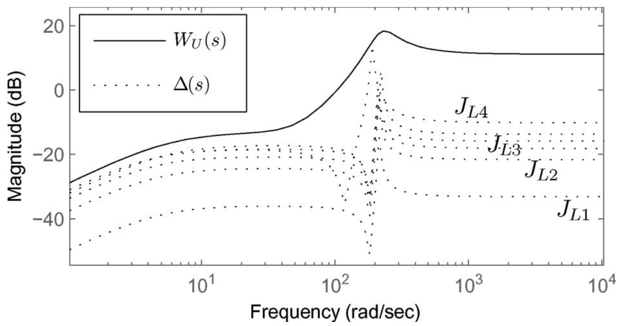

Fig. 8. Responses of $\Delta(s)$ for $J_{L1}=0.0035 \sim J_{L4}=0.007$ ($K_f=75.5$).

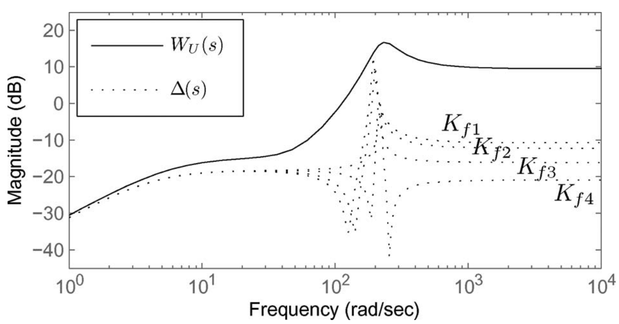

Fig. 9. Responses of $\Delta(s)$ for $K_{f1} = 62 \sim K_{f4} = 82$ ($J_L = 0.007$).

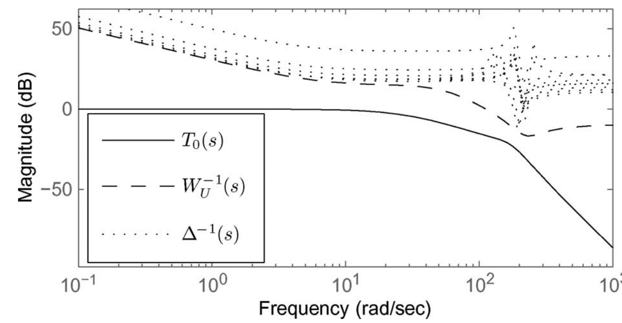

Fig. 10. Test of general robust stability condition (23) to $\Delta(s)$ for variations of $J_L = 0.0035 \sim 0.007$ and $K_f = 58 \sim 82$.

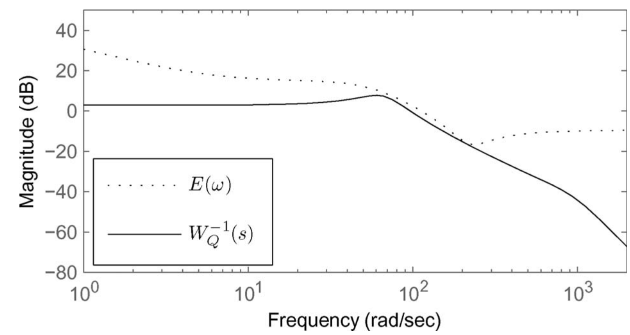

Fig. 11. Test of inequality (22) by frequency responses of $E(\omega)$ and $W_Q^{-1}(s)$.

its transient time to enter error range of 0.05 is 0.4 s. From (2) and Table I, it is clear that all the roots of

$$\begin{aligned} M_{n2}(s) = & J_{Mn}J_{Ln}s^3 + (J_{Mn}B_{Ln} + J_{Ln}B_{Mn})s^2 \\ & + [K_{fn}(J_{Mn} + J_{Ln}) + B_{Mn}B_{Ln}]\, s \\ & + K_{fn}(B_{Mn} + B_{Ln}) \end{aligned} \tag{30}$$

are placed on half left plan, where parameters suffixed by n indicate the nominal values and $J_{Ln} = 0.0055$, $K_{fn} = 75$. Thus, feedback controller designed according to (5)–(7)

$$C(s) = \frac{N_{C1}(s)M_{n2}(s)}{M_{C1}(s)} \tag{31}$$

is proper and will never lead to unstable pole-zero cancellation. In (31), $N_{C1}(s) = 1$, and $M_{C1}(s) = K_f\sigma_M[(\sigma_M s)^3 + c_1(\sigma_M s)^2 + c_2(\sigma_M s) + c_3]$ resulted from $G_{yr}(s) = G_M(s)$.

2) DOB Design: In order to design DOB according to the method of Section III, it needs to consider model perturbation due to parameter variations and uncertainties. Figs. 8 and 9 show maximum limits of relative model variation $\Delta(s) = (P(s) - P_n(s))/P_n(s)$ and illustrate that uncertainty of J_L and K_f produce sensitive magnitude peak's variation of $\Delta(s)$. From the figures, it is verified that a function

$$W_U(s) = \frac{3s(s^2 + 78s + 3000)}{(s+6)(s^2 + 100s + 5000)} \tag{32}$$

can be used as an upper limit function of all model perturbations, i.e., $|W_U(j\omega)| > |\Delta(j\omega)|$. As shown in Fig. 10, feedback controller (31) satisfies the general robust stability sufficient condition (23) without DOB.

Considering the nominal model's relative order $k = 4$ and internal model order demand $q = 2$, the orders of Q-filter are selected as $n = 5$, $m = 1$ [24]. Thus, a nonproper weighting function $W_Q(s)$ that satisfies these order restrictions and (22) can be selected as

$$W_Q(s) = \frac{1}{6.17 \times 10^9}(s^2 + 40s + 4.44 \times 10^3) \times (s^2 + 1.0 \times 10^3 s + 6.0 \times 10^5). \tag{33}$$

The weighting function for optimal disturbance suppression performance at low frequencies is

$$W_C(s) = \frac{s^2 + 10s + 4600}{1000(s+\lambda)^2} \tag{34}$$

where $\lambda = 0.001$ is a sufficiently small value for satisfying the preconditions of standard H_∞ control framework for internal model order of the Q-filter. From (33) and (34), it can be seen that the inverses of $W_Q(s)$ and $W_C(s)$ have comparatively large peak magnitudes around cutoff frequency. This property is helpful to lower sensitivity at low frequencies according to loop shaping theory. From Fig. 11, it is verified that (22) is satisfied by $W_Q(s)$ and $E(\omega)$. The Q-filter design problem (28) with weighting functions (33) and (34) can be systematically solved by the procedure proposed in [24] without any numerical computation with respect to Q-filter's coefficients. The optimal Q-filter satisfying structural restricts $n = 5$, $k = 4$, and $q = 1$ is obtained at $\gamma_{\max} = 304.3633$ as follows:

$$Q_{\text{opt}}(s) = \frac{\sum_{j=0}^{1} b_j \cdot s^j}{\sum_{i=0}^{5} a_i \cdot s^i} \tag{35}$$

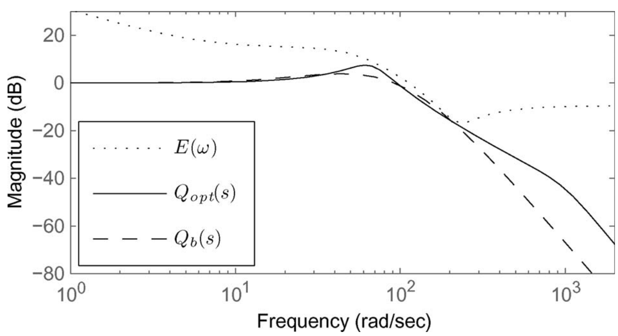

Fig. 12. Frequency responses of designed optimal $Q_{\mathrm{opt}}(s)$ and binomial coefficient model $Q_b(s)$ with $\sigma_b = 0.0102$.

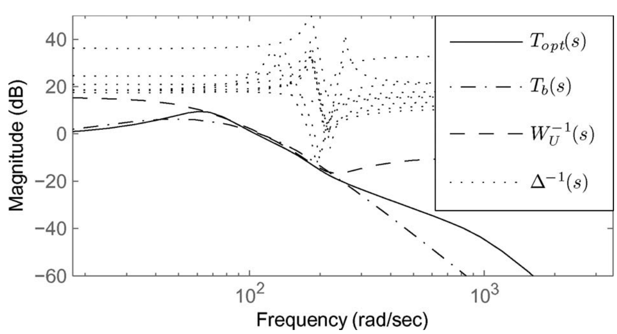

Fig. 13. Test of (12) using $Q_{\mathrm{opt}}(s)$ and $Q_b(s)$ with $\sigma_b = 0.0102$.

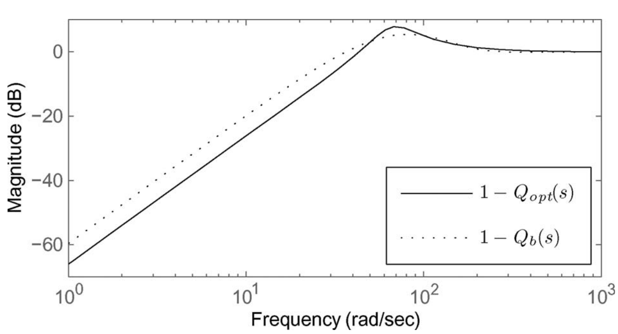

Fig. 14. Comparison of disturbance suppressions at low frequencies.

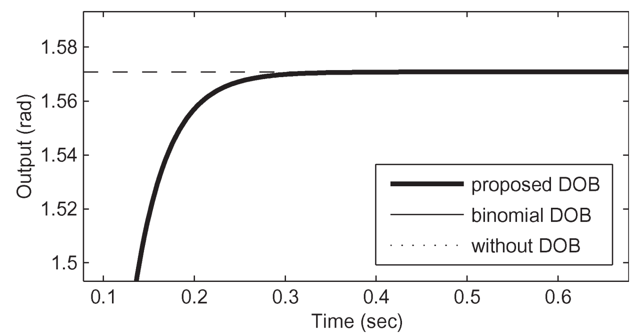

Fig. 15. Responses to rectangle wave reference with all parameter matched.

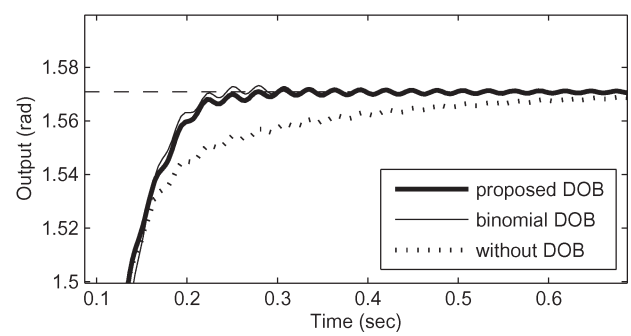

Fig. 16. Time responses for step reference ($J_L = 0.005$, $K_f = 82$).

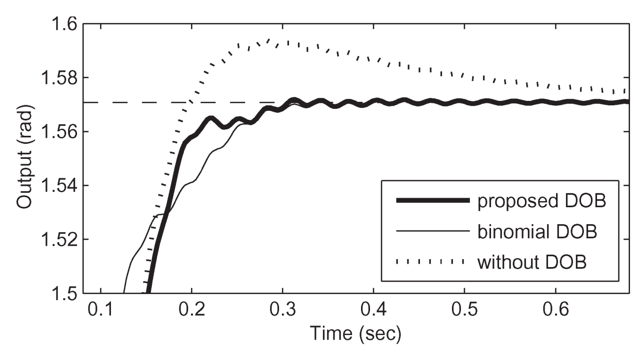

Fig. 17. Time responses for step reference ($J_L = 0.007$, $K_f = 75$).

where $a_5 = 1,\ a_4 = 1072,\ a_3 = 1.078 \times 10^6,\ a_2 = 7.949 \times 10^7,\ \ a_1 = b_1 = 5.92 \times 10^9,$ and $a_0 = b_0 = 1.591 \times 10^{11}$. Fig. 12 shows the frequency responses of the designed $Q_{\mathrm{opt}}(s)$ and binomial coefficient model $Q_b(s) = (5\sigma_b s + 1)/(\sigma_b s + 1)^5$, which is commonly used as conventional low-pass filter of DOB, and $\sigma_b = 0.0102$ gives about the same robust stability margin as $Q_{\mathrm{opt}}(s)$. The designed $Q_{\mathrm{opt}}(s)$ is verified to satisfy the robust stability sufficient condition (13) to variations and uncertainties of parameters in Fig. 13, where $T_{\mathrm{opt}}(s)$ and $T_b(s)$ are complementary sensitivity functions (10) of closed-loop systems using $Q_{\mathrm{opt}}(s)$ and $Q_b(s)$ separately. From the figure, it can be known that $T_{\mathrm{opt}}(s)$ has larger robust stability margin against variations of parameters than $T_b(s)$. Using $Q_b(s)$, increase of time constant σ_b can lead to improvement of this margin. However, it also leads to more degradation of disturbance suppression performance. In Fig. 14, this performance is evaluated by $1 - Q(s)$ as shown in (15)–(17). The results show that $1 - Q_{\mathrm{opt}}(s)$ is about 7 dB (approximately 2.2 times) lower than $1 - Q_b(s)$ with $\sigma_b = 0.0102$ at frequencies of $0 \sim 40$ rad/s. Though $Q_{\mathrm{opt}}(s)$'s peak magnitude around cutoff frequency is somehow higher than that of Q_b, frequency of disturbance is so low that its power around the cutoff frequencies can be neglected in most application practices. In case that disturbance power at these frequencies is not negligible, we can slightly lower the peak magnitude permitted by the weighting functions in (33) and (34) so as to reduce the effect of the disturbances at these frequencies on output, in design steps.

The design results illustrated above show that the proposed method provides better disturbance suppression performance under the same robust stability condition.

3) Analysis of the Experimental Results: Fig. 15 shows the time responses of the control systems to step reference with set point of $\pi/2$, where all the parameters of the plant and nominal model are matched and there are no exotic disturbance. It illustrates that the controller (31) provides the required response characters with nonvibration and nonovershoot by canceling the oscillation modes of the plant. Figs. 16 and 17 show the time

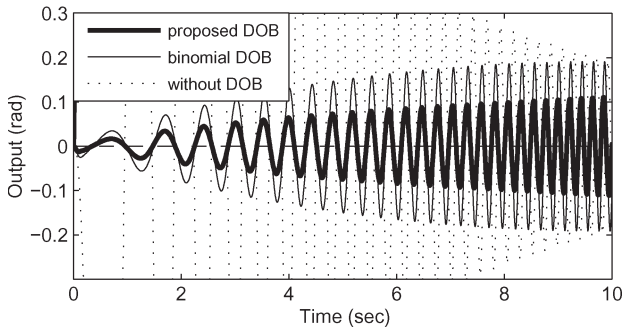

Fig. 18. Responses for chirp disturbance generated by load motor.

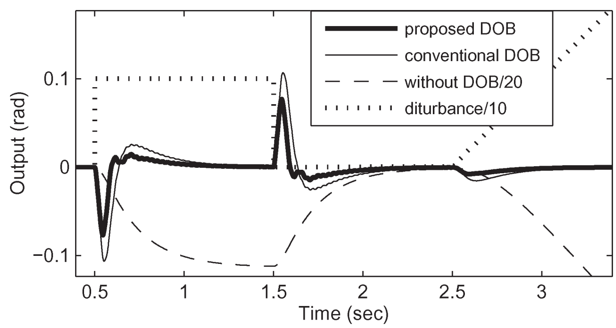

Fig. 19. Responses to square disturbance generated by load motor.

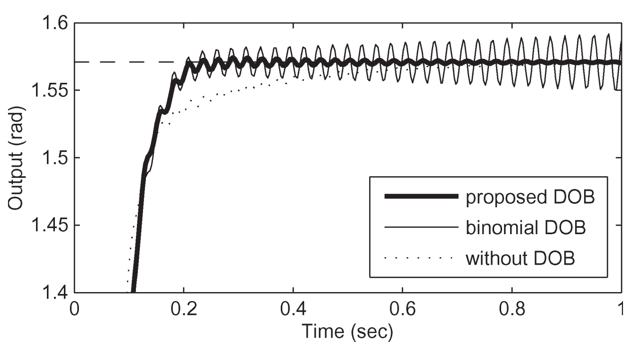

Fig. 20. Responses to step reference ($J_L = 0.0045$, $K_f = 75.5$).

responses in the case of $J_L = 0.005$, $K_f = 82$, and $J_L = 0.007$, $K_f = 75$, where oscillations appear, but they are so slight that can be negligible. Except that the responses without DOB have considerable errors sensitive to variations of parameters, all the responses with two DOBs for allowed variations of J_L and K_f are similar, which coincides with analysis of Fig. 13, indicating both DOBs have the same robustness. However, the essential difference between two DOBs appears in performances of suppressing low frequency disturbance. The responses to chirp type load torque with frequency scope of 0 $\sim$ 50 rad/s and square load torque generated by load motor are shown in Figs. 18 and 19, where error magnitudes by $Q_{\rm opt}(s)$ is about 2 times less than that by $Q_b(s)$, which coincides with design result shown in Fig. 14. To lower σ_b of $Q_b(s)$ to $\sigma_b = 0.0072$ in order to coincide the performance of suppressing low frequency disturbance with that of $Q_{\rm opt}(s)$ results in degradation of stability of closed-loop system as shown in Fig. 20. In particular, in the case that the parameter

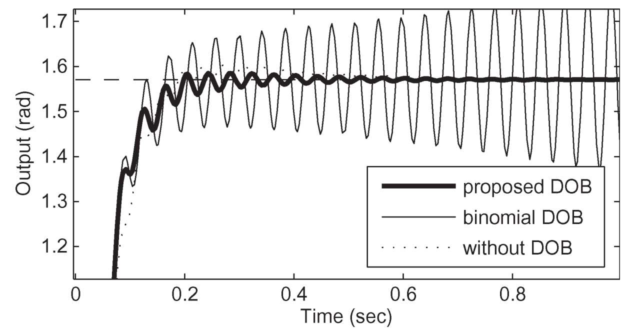

Fig. 21. Responses to step reference ($J_L = 0.0068$, $K_f = 50$).

values go beyond the allowed scopes, proposed DOB still can keep robustness to some extend, while conventional one gets unstable, as shown in Fig. 21.

B. Case of Using Sensor Mounted on Driving Shaft

In design of control using sensor of motor side, $G_{MM}(s)$ of (3) is used as model of plant for robust stability analysis in Section III. In this case, the restrictions to design gets more rigorous because the output to be controlled is not measured.

1) Feedback Controller: The transfer function from $y = \theta_M$ to load side output $y_o = \theta_L$ is

$$G_{y_o y}(s) = \frac{b_{G0}}{b_{G2}s^2 + b_{G1}s + b_{G0}} = \frac{K_f}{J_L s^2 + B_L s + K_f}. \tag{36}$$

In order to remove this oscillation modes from transfer function $G_{y_o r}(s)$ from r to y_o, numerator of $G_{yr}(s)$ of (7) should include the polynomial $b_{G2}s^2 + b_{G1}s + b_{G0}$, i.e., $N_{C1} = b_{G2}s^2 + b_{G1}s + b_{G0}$ because $G_{y_o r}(s) = G_{y_o y}(s)G_{yr}(s)$. From (6), it is clear that the third $M_{C1}(s)$ can sufficiently make $C(s)$ to be proper, since $N_{C1}(s) = N_{n2}(s)$. Then, from (5)–(7), controller

$$C(s) = \frac{M_{n2}(s)}{M_{C1}(s)} \tag{37}$$

is proper and can cancel all the stable resonance and inverse resonance modes of the plant in nominal state. In (37), $M_{n2}(s)$ is defined in (30), and

$$M_{C1}(s) = K_{fn}\sigma_M^4 s^3 + K_{fn}\sigma_M^3 c_1 s^2 + \left(K_{fn}\sigma_M^2 c_2 - J_{Ln}\right)s + K_{fn}\sigma_M c_3 - B_{Ln} \tag{38}$$

resulted from $G_{y_o r}(s) = G_M(s)$ where $G_M(s)$ is the reference model shown in (29).

2) Design of DOB: Parameter variations are the same as above. Fig. 22 shows the transfer functions $G_{MM}(s)$ which are regarded as plant model of DOB for the parameter variations. A function

$$W_U(s) = \frac{4.2s(s^2 + 30s + 1200)}{(s + 10)(s^2 + 30s + 15\,000)} \tag{39}$$

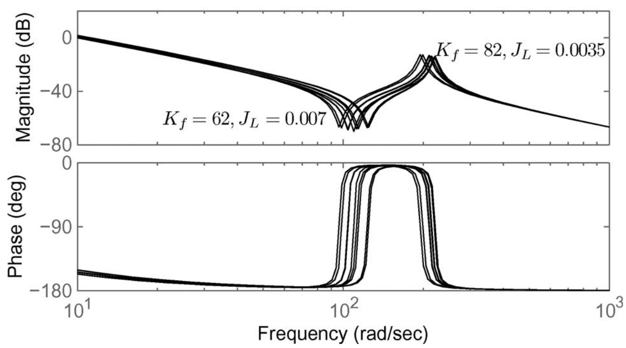

Fig. 22. Frequency responses of $G_{MM}(s)$ for different J_L and K_f.

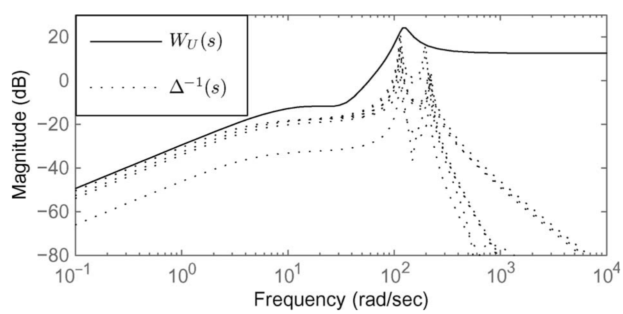

Fig. 23. Model perturbations and its upper limit function.

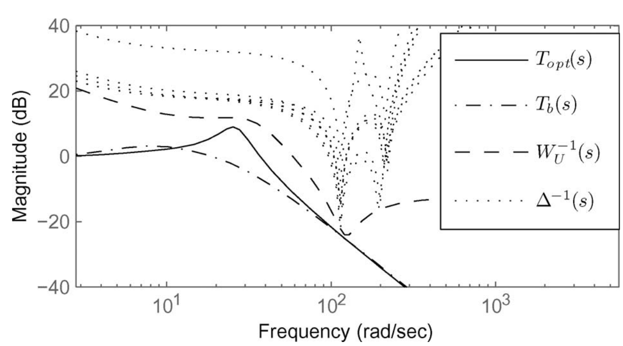

Fig. 24. Test of robust stability of closed-loop system.

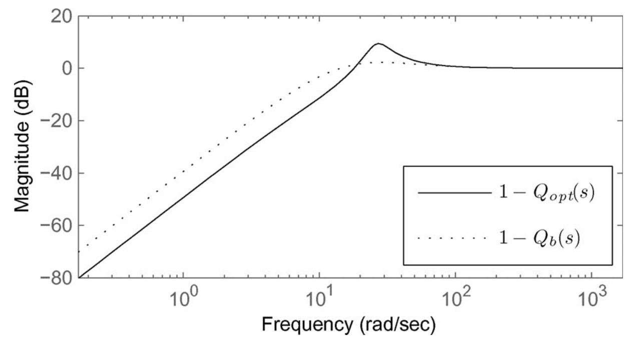

Fig. 25. Comparison of sensitivity functions at low frequencies.

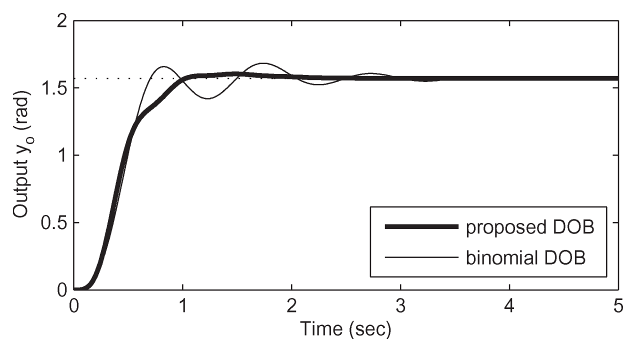

Fig. 26. Responses to step reference with $J_L = 0.025$.

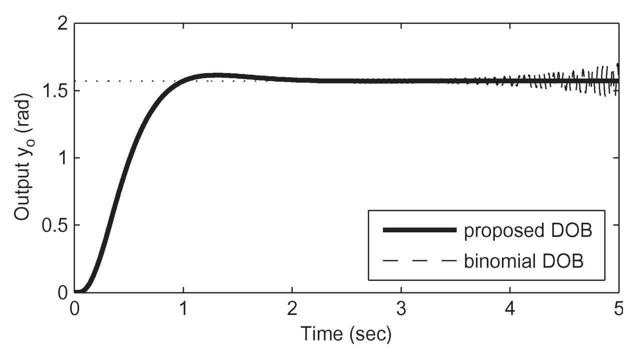

Fig. 27. Responses to step reference with $K_f = 36.5$.

can be selected as a upper limit function of all $\Delta(s)$ as shown in Fig. 23. Considering relative order $k = 2$ (equal to the order of $G_{MM}(s)$) and order demand $n = 3$, weighting functions satisfying (22) and (26) can be selected as

$$W_Q(s) = \frac{1}{780}(s^2 + 9s + 676) \tag{40}$$

$$W_C(s) = \frac{s^2 + 8s + 2800}{1000(s+\lambda)^2}. \tag{41}$$

By the same procedure as above, the optimal Q-filter is obtained as

$$Q_{\text{opt}}(s) = \frac{779.6s + 5299}{s^3 + 18.08s^2 + 779.6s + 5299}. \tag{42}$$

Fig. 24 shows that $Q_{\text{opt}}(s)$ satisfies the robust stability condition (11) and (13). For comparison, in Figs. 24 and 25, conventional Q-filter $Q_b(s)$ with $\sigma_b = 0.06$ which has the same robust stability margin as $Q_{\text{opt}}(s)$ is plotted together. Design result indicates that the proposed DOB provides much less sensitivity than the conventional one as shown in Fig. 25, meaning that its performance of suppressing disturbance at low frequencies outweighs that of $Q_b(s)$ by 12 dB (about 3 $\sim$ 4 times).

3) Analysis of the Experiment Results: In allowed scope of parameter variations, control systems with both DOBs, except one without DOB, give the same response results as previous section, so omitted here. Figs. 26 and 27 show the step responses in the extreme case that J_L and K_f go far from allowed scopes separately. As shown in Fig. 26, when the load wheels with large inertia are added, the response with conventional DOB produces considerable vibration of low frequency and low damping rate, while that of the proposed DOB keeps better performance of suppressing vibration. The results of Fig. 27 show that decrease of elasticity of torsion shaft leads to loss

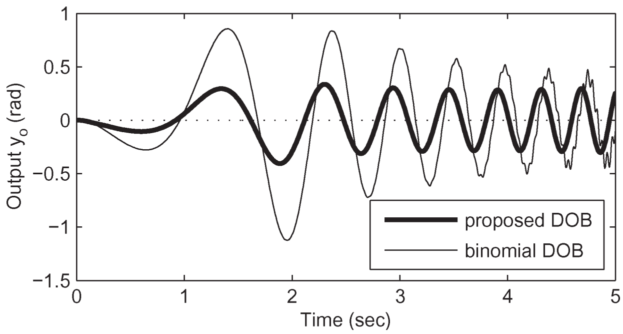

Fig. 28. Responses to chirp disturbance generated by load motor.

of stability in the response with conventional DOB, while that of the proposed DOB maintains stable. The responses to chirp disturbance generated by load motor with frequency scope of $0 \sim 20$ rad/s are shown in Fig. 28, the difference between which coincides with design results shown in Fig. 25. The same results of responses to square type load torque as one in Section IV-A are obtained, but omitted here. Decrease of σ_b of $Q_b(s)$ to coincide sensitivity with $Q_{\rm opt}(s)$ at low frequencies results in loss of robust stability to parameter variation like the previous case shown in Section IV-A, omitted here.

Two design cases and experimental results show that the proposed design method of robust DOB provides good robustness in spite of parameter variations and the high performance of suppressing disturbance in two-inertia system.

V. Conclusion

This paper proposes a vibration suppression control design methods of two-inertia system based on DOB. The design of feedback controller to cancel oscillation modes of two-inertia system can achieve smooth dynamic response of nonvibration and nonovershoot. This performance, however, cannot be maintained because of the existence of model perturbation due to variations and uncertainties of the plant's parameters and exotic disturbance. The shortcoming can be compensated by DOB. The proposed method for designing Q-filter of DOB gives systematic and straightforward procedure with combination of a new robust stability condition of DOB system and standard H_∞ control framework. In this method, a new robust stability sufficient condition is developed and adopted to select weighting function for H_∞ design so that the DOB design problem can be transformed into standard H_∞ control framework. Since Q-filter is designed with weighting functions carefully selected in consideration of detailed system characteristics, it can achieve better disturbance suppression performance than conventional Q-filter model such as binomial coefficient model. The design and experiments illustrate that the proposed method can improve the disturbance suppression performance and robustness of the system.

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