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–Brief Paper–

ROBUST OPTIMAL DISTURBANCE OBSERVER DESIGN FOR THE NON-MINIMUM PHASE SYSTEM

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ABSTRACT

In this paper, a design strategy of robust disturbance observer is proposed systematically for stable non-minimum phase systems. This strategy synthesizes the internal and robust stability, relative order and mixed sensitivity design requirements together to establish the optimization function. The optimal solution is obtained by standard H_∞ control theory under the condition of guarantying the presented requirements. Simulation results of a rotary mechanical system show the effectiveness of the proposed strategy.

Key Words: Non-minimum phase system, disturbance observer, internal and robust stability, H_∞ control theory.

I. INTRODUCTION

In most practical industrial processes, the inevitable system uncertainties and external disturbances will have great influence on the performance of the control system. The disturbance observer based (DOB) control scheme was originally proposed by Ohnishi in 1987 [1], and it's effectiveness in disturbance rejections has been shown in many applications [2–5].

The design of the Q filter is the key point of DOB configuration, and has attracted much attention from researchers. There have been abundant results in using the H_∞ control theory for Q filter design [6–8]. Linear matrix inequalities (LMI) or algebraic Riccati equations were applied in [6,7] to optimize the Q filter with static gain. The standard H_∞ scheme is employed in [8] to optimize the Q filter. However, this research neglects the internal stability of the system, and cannot be used directly in non-minimum phase (NMP) systems. Since the inverse of the nominal plant is required in DOB configuration, the internal stability problem occurs if the nominal plant has the right-half of the s-plane (RHP) zeros.

In this paper, the DOB configuration for an NMP system is investigated. We first consider the internal and robust stability, relative order, mixed sensitivity design requirements together to establish an optimization function. Then, the optimization problem is transformed into a standard H_∞ one, based on which the solution of the Q filter is optimized by the existing standard H_∞ control theory. Finally, a design example is presented specifically on a mechanical system to show the effectiveness of the proposed strategy.

II. PROBLEM STATEMENT

The traditional control system based on DOB is expressed in Fig. 1, where $P(s)$ is the plant model, $P_n(s)$ is the nominal model and, $Q(s)$ is the Q filter to be designed. $U_r(s)$, $Y(s)$, $D(s)$ and $N(s)$ denote the Laplace transformation of reference input u_r, output y, external disturbances d and measurement noise n, respectively. We focus on the DOB configuration for stable NMP systems. The system model $P(s)$ is described with multiplicative uncertainty as:

$$P(s) = P_n(s)(1 + \Delta(s)), \tag{1}$$

where $P(s)$ and $P_n(s)$ are all stable plants with RHP zeros. The nominal plant $P_n(s)$ is expressed as:

$$P_n(s) = \frac{N(s)}{D(s)} \prod_i (-s + \xi_i), \tag{2}$$

where $Re(\xi_i) > 0$, $N(s)$ and $D(s)$ have no root with positive real part.

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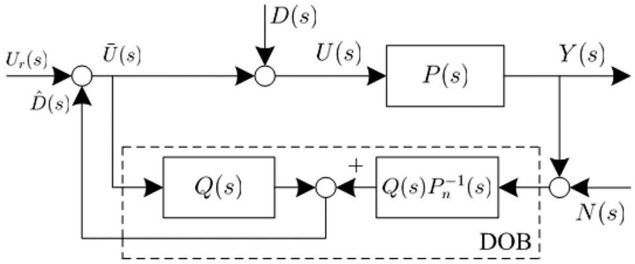

Fig. 1. The DOB based structure.

III. ROBUST DISTURBANCE OBSERVER DESIGN

3.1 Internal stability constraint

A control system is internally stable if bounded signals injected at any point of the control system generate bounded responses at any other point [10]. The internal stability is a basic requirement for a practical closed-loop system. Here, we give the sufficient condition of the internal stability for DOB structure.

Theorem 1. The control system described in Fig. 1 is internally stable if the following requirements are satisfied:

Requirement 1. $Q(s) \in \mathcal{RH}_\infty$ and it can eliminate all the RHP poles of $P_n^{-1}(s)$.

Requirement 2. The open-loop transfer function from U_r to U is stable.

Requirement 3. The robust stability against the system uncertainties should be satisfied.

Proof. The six transfer function from $\begin{bmatrix} U_r & D & N \end{bmatrix}^{\mathrm{T}}$ to $[Y \quad U]^{\mathrm{T}}$ are given as:

$$\frac{1}{M}\begin{bmatrix} P(s) & P(s)(1-Q(s)) & -P_n^{-1}(s)P(s)Q(s) \\ 1 & (1-Q(s)) & -P_n^{-1}(s)Q(s) \end{bmatrix}, \quad (3)$$

where $M = 1 + P_n^{-1}(s)P(s)Q(s) - Q(s)$. If all the components of the matrix in (3) and $\frac{1}{M(s)}$ are all in $\mathcal{RH}_\infty$, then the transfer function from $\begin{bmatrix} U_r & D & N \end{bmatrix}^{\mathrm{T}}$ to $[Y \quad U]^{\mathrm{T}}$ is stable, based on which we can guarantee the internal stability.

Since $P(s)$ and $P_n(s)$ are with no RHP poles, if Requirement 1 is satisfied, then we can conclude that all components of matrix in (3) are in $\mathcal{RH}_\infty$.

Noticing that $\frac{1}{M(s)}$ is the transfer function from U_r to U. If the open-loop transfer function from U_r to U is stable and the closed-loop system is robustly stable, then the closed-loop system is input output stable with input U_r and output U. Hence, we notice that $\frac{1}{M(s)} \in \mathcal{RH}_\infty$ if Requirements 2 and 3 are satisfied. It is clear that the control system described in Fig. 1 is internally stable if Requirements 1 to 3 are satisfied.

3.2 Robust DOB design

The transfer function of the DOB structure can be expressed as:

$$\begin{aligned} Y(s) = & M^{-1}(s)\left[P(s)(1-Q(s))D(s) + P(s)U_r(s) \right. \\ & \left. -P_n^{-1}(s)P(s)Q(s)N(s)\right]. \end{aligned} \quad (4)$$

We define a set of $Q(s)$ as:

$$\Omega_k = \left\{ F(s) | F(s) = \frac{A(s)}{B(s)} = \frac{\sum_{j=0}^{q} a_j s^j}{\sum_{i=0}^{p}, b_i s^i}, \right. \\ \left. a_q \neq 0, b_p \neq 0, p - q \geq k \right\}, \quad (5)$$

where k is the relative order of $P_n(s)$, $A(s)$ and $B(s)$ are coprime polynomials.

We hope the solution of the Q filter can eliminate the disturbances as far as possible under the condition of guarantying the requirements in Theorem 1. From (4), the disturbance and measurement noise attenuation problem can be regarded as selection of the tradeoff between the sensitivity and complementary sensitivity functions:

$$\min_{Q(s)} |W_1(j\omega)(1-Q(j\omega))|, \min_{Q(s)} |W_N(j\omega)Q(j\omega)|, \quad (6)$$

where W_1 and W_N are the weighting functions which reflect the prior frequency property of the external disturbances and measurement noise, respectively. Then we analyze the robust stability of the closed-loop system. According to the small gain theory, we get the requirement of robust stability as [2]:

$$\|Q(s)\Delta(s)\|_\infty < 1. \quad (7)$$

With the consideration of the internal stability of the closed-loop system, we combine the requirements in (6) and (7) together to acquire the new optimization function as:

$$\max \gamma,$$
$$\text{s.t.} \min_{\substack{Q(s)\in\Omega_k \\ Q(s)\in RH_\infty}} \left\| \begin{bmatrix} \gamma W_1(s)(1-Q(s)) \\ W_2(s)Q(s) \end{bmatrix} \right\|_\infty < 1, \tag{8}$$

where $|W_2(j\omega)| = \max\{|\Delta(j\omega)|, |W_N(j\omega)|\}, \forall\omega$. $|W_2(j\omega)|$ should be selected as close as possible to $|\Delta(j\omega)|$ and $|W_N(j\omega)|$, otherwise it will lead to the system being more conservative.

Now, we present the optimized solution for Q filter. For the optimization problem in (8), it is very hard to get the solution $Q(s)$ directly. The standard H_∞ control theory is employed for this kind of optimization problem in [8]. Defining the transfer function of the virtual loop as: $\tilde{L}(s) = \frac{Q(s)}{1-Q(s)} = \tilde{P}(s)\tilde{K}(s)$, the Q filter design problem becomes a standard H_∞ problem as:

$$\max \gamma,$$
$$\text{s.t.} \min_{\substack{Q(s)\in\Omega_k \\ Q(s)\in RH_\infty}} \left\| \begin{bmatrix} \gamma W_1(s)\left(I+\tilde{P}\tilde{K}\right)^{-1} \\ W_2(s)\tilde{P}\tilde{K}\left(I+\tilde{P}\tilde{K}\right)^{-1} \end{bmatrix} \right\|_\infty < 1, \tag{9}$$

where $\tilde{L}(s) = \tilde{P}(s)\tilde{K}(s)$ and $\tilde{P}(s), \tilde{K}(s)$ are the virtually controlled objective and controller, respectively.

The virtually controlled objective $\tilde{P}(s)$ is given as:

$$\tilde{P}(s) = P_0(s)P_A(s), \tag{10}$$

where $P_0(s)$ is a stable plant and $W_2(s)P_0(s)$ must be proper to guarantee the solution of the standard H_∞ problem. $P_A(s)$ is an allpass portion which includes all the RHP zeros of $P_n(s)$:

$$P_A(s) = \prod_i \frac{-s+\xi_i}{s+\xi_i^H}, Re(\xi_i) > 0, \tag{11}$$

where the superscript H denotes the complex conjugate.

For a given virtually controlled objective $\tilde{P}(s)$, if we can acquire the optimal solution of the virtual controller $\tilde{K}(s)$, then we can obtain the Q filter as:

$$Q(s) = \frac{\tilde{P}(s)\tilde{K}(s)}{1+\tilde{P}(s)\tilde{K}(s)}. \tag{12}$$

Theorem 2. The optimized Q filter has the following properties:

1. The relative order of optimized $Q(s)$ is higher than or equal to that of weighting function $W_2(s)$.
2. The optimized $Q(s) \in \mathcal{RH}_\infty$, and it has all the RHP zeros of $P_n(s)$.
3. The open-loop transfer function from U_r to U is stable.

Proof.

1. From the optimization function in Eq. (8) and loop shaping theory, we know that $\|1-Q(s)\|_\infty < \left\|\frac{1}{\gamma W_1(s)}\right\|_\infty$ and $\|Q(s)\|_\infty < \left\|\frac{1}{W_2(s)}\right\|_\infty$. Then we can obtain the following equation as:

$$\lim_{\omega\to\infty} |W_2(j\omega)Q(j\omega)| < 1, \tag{13}$$

hence, the relative order of $Q(s)$ is higher than or equal to that of $W_2(s)$.

2. From the description of H_∞ optimal control problem, we know that the closed-loop system of the virtual H_∞ control problem is internally stable, then $Q(s) = \frac{\tilde{P}(s)\tilde{K}(s)}{1+\tilde{P}(s)\tilde{K}(s)} \in \mathcal{RH}_\infty$.

   Since there is no RHP zero-pole cancellation between virtual plant and controller, $\tilde{L}(s)$ has all the RHP zeros of $\tilde{P}(s)$. Assume that $\tilde{L}(s) = \frac{N_{L-}(s)N_{L+}(s)}{D_L(s)}$, and $D_L(s)$ is coprime with $N_{L-}(s)N_{L+}(s)$, where $N_{L+}(s) = \prod_i(-s+\xi_i), Re(\xi_i) > 0$. Then, $Q(s)$ can be expressed as:

$$Q(s) = \frac{N_{L-}(s)N_{L+}(s)}{D_L(s)+N_{L-}(s)N_{L+}(s)}, \tag{14}$$

   that is, the optimized $Q(s)$ can eliminate all the RHP poles of $P_n^{-1}(s)$.

3. The open-loop transfer function from U_r to U is give as: $G_{UU_r} = \frac{1}{1-Q(s)}$. According to Eq. (12), this transfer function can be rewritten as: $G_{UU_r} = 1 + \tilde{P}(s)\tilde{K}(s)$. Noticing the virtual plant and controller are all stable, hence, the transfer function G_{UU_r} is stable.

The above Theorem shows that if the weighting functions and virtual control objective are well-selected, the optimized Q filter satisfies the order constraint, and can eliminate the unstable poles of $P_n^{-1}(s)$.

IV. APPLICATION OF MECHANICAL SYSTEM

A rotary mechanical system is applied to verify the effectiveness of the proposed strategy. The continuous-time plant transfer function is [11]

$$P(s) = \frac{123.853 \times 10^4(-s+3.5)}{(s^2+6.5s+42.25)(s+45)(s+190)}. \quad (15)$$

Noticing that the poles $s = -45$ and $s = -190$ are far away from the dominant conjugate poles, we select the nominal model as:

$$P_n(s) = \frac{144.86(-s+3)}{(s^2+6.5s+42.25)}. \quad (16)$$

To suppress the constantly external disturbances as well as guarantee the robust stability, we select the following weighting functions:

$$W_1(s) = \frac{1}{s}, W_2(s) = \frac{0.2s+3}{9}. \quad (17)$$

Noticing that the relative order of $P_n(s)$ is 1, the relative order of $W_2(s)$ is chosen as 1 to make the relative order of $Q(s)$ higher than or at least equal to that of $P_n(s)$. Then, the virtual control objective and optimized virtual controller are given as:

$$\tilde{P}(s) = \frac{1}{s+1} \cdot \frac{-s+3}{s+3}, \tilde{K}(s) = \frac{45(s+3)(s+1)}{s(s+64.57)}, \quad (18)$$

and the Q filter is expressed as:

$$Q(s) = \frac{45(-s+3)}{s^2+19.57s+135}. \quad (19)$$

Then, a prefilter is selected as:

$$\frac{s^2+6.5s+42.25}{144.86(0.2s^2+1.6s+3)}. \quad (20)$$

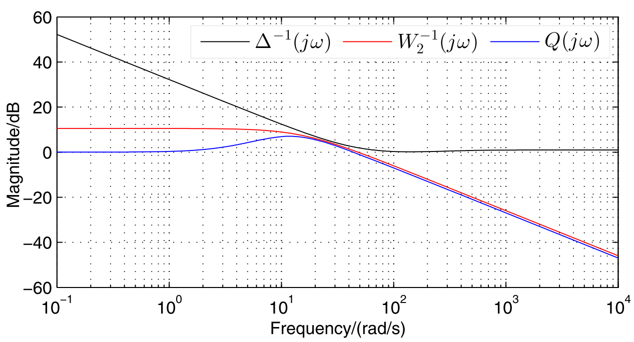

Fig. 2. Verification of robust stability condition.

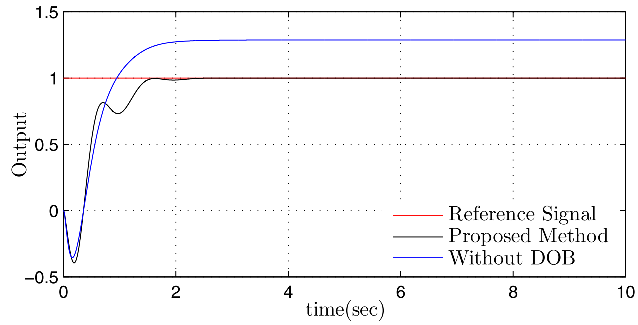

Fig. 3. Step response with constant disturbances.

Fig. 2 shows the robust stability condition of the closed-loop system with DOB. The weighting function that reflects the robust stability condition $W_2(s)$ is well-selected. It is also verified that the optimized Q filter satisfies the robust stability condition very well. Fig. 3 shows the control performance of control system in the presence of constant disturbance. Without DOB, there exists steady-state error caused by external disturbances and system uncertainties. The designed DOB can eliminate the steady-state error successfully.

V. CONCLUSIONS

This paper proposes a systematic DOB design strategy for stable NMP system. An optimization function is established based on several design requirements, based on which standard H_∞ control theory is employed to obtain the optimal solution. Simulations are carried out on a rotary mechanical system with a RHP equal to zero. It is verified that the proposed DOB can be employed successfully to suppress the influence caused by disturbances. The robust stability is also guaranteed against internal uncertainties.

REFERENCES

1. Ohnishi, K., "A new servo method in mechatronics," *Trans. Jpn. Soc. Elect. Eng.*, Vol. 107-D, pp. 83–86 (1987).
2. Xing, D., J. Su, Y. Liu, and J. Zhong, "Robust approach for humanoid joint control based on a disturbance observer," *IET Contr. Theory Appl.*, Vol. 5, No. 14, pp. 1630–1636 (2011).
3. Yang, K., Y. Choi, and W. K. Chung, "On the tracking performance improvement of optical disk drive servo systems using error-based disturbance

observer," *IEEE Trans. Ind. Electron.*, Vol. 52, No. 1, pp. 270–279 (2005).
4. Na, J., R. Costa-Castello, R. Grino, and X. Ren, "Discrete-time repetitive controller for time-delay systems with disturbance observer," *Asian J. Control*, Vol. 14, No. 5, pp. 1340–1354 (2012).
5. Chen, X. S., J. Yang, S. H. Li, and Q. Li, "Disturbance observer based multi-variable control of ball mill grinding circuits," *J. Process Control*, Vol. 19, No. 7, pp. 1205–1213 (2009).
6. Wang, C. C. and M. Tomizuka, "Design of robustly stable disturbance observers based on closed loop consideration using H_∞ optimization and its application to motion control systems," *Proc. Amer. Control Conf.*, IEEE, Boston, MA, USA, pp. 3764–3769 (2004).
7. Thum, C. K., C. Du, F. L. Lewis, B. M. Chen, and E. H. Ong, "H_∞ disturbance observer design for high precision track following in hard disk drives," *IET Contr. Theory Appl.*, Vol. 3, No. 12, pp. 1591–1598 (2009).
8. Yun, J. N., J. Su, Y. I. Kim, and Y. C. Kim, "Robust distuabance observer for two-inertia system," *IEEE Trans. Ind. Electron*, Vol. 60, No. 7, pp. 2700–2710 (2013).
9. Shim, H. and N. H. Jo, "An almost necessary and sufficient condition for robust stability of closed-loop systems with disturbance observer," *Automatica*, Vol. 45, No. 1, pp. 296–299 (2009).
10. Morari, M. and E. Zafiriou, *Robust Process Control*, Prentice Hall, Englewood Cliffs, NJ (1989).
11. Freeman, C. T., P. L. Lewin, and E. Rogers, "Experimental evaluation of iterative learning control algorithms for non-minimum phase plant," *Int. J. Control*, Vol. 78, No. 11, pp. 826–846 (2005).