ME 465 – Fall 2001 Homework #3 - Solution

1. Fill in the table of Denavit-Hartenburg parameters for the three-link RPR robot shown below.

Joint	q_I	d_i	a_i	α_i
1	θ_1	0	a_1	270°
2	0	d_2	0	90°
3	θ_2	0	a_3	0°

2. Use the results from the table above and the D-H matrix given on page 18 of your notes to write the three Denavit-Hartenburg transformation matrices (one for each joint) for the RPR Planar Robot (shown below)

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & 0 & -\sin q_1 & a_1\cos q_1 \\ \sin q_1 & 0 & \cos q_1 & a_1\sin q_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \underline{A}_1 \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \underline{A}_2 \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix}$$

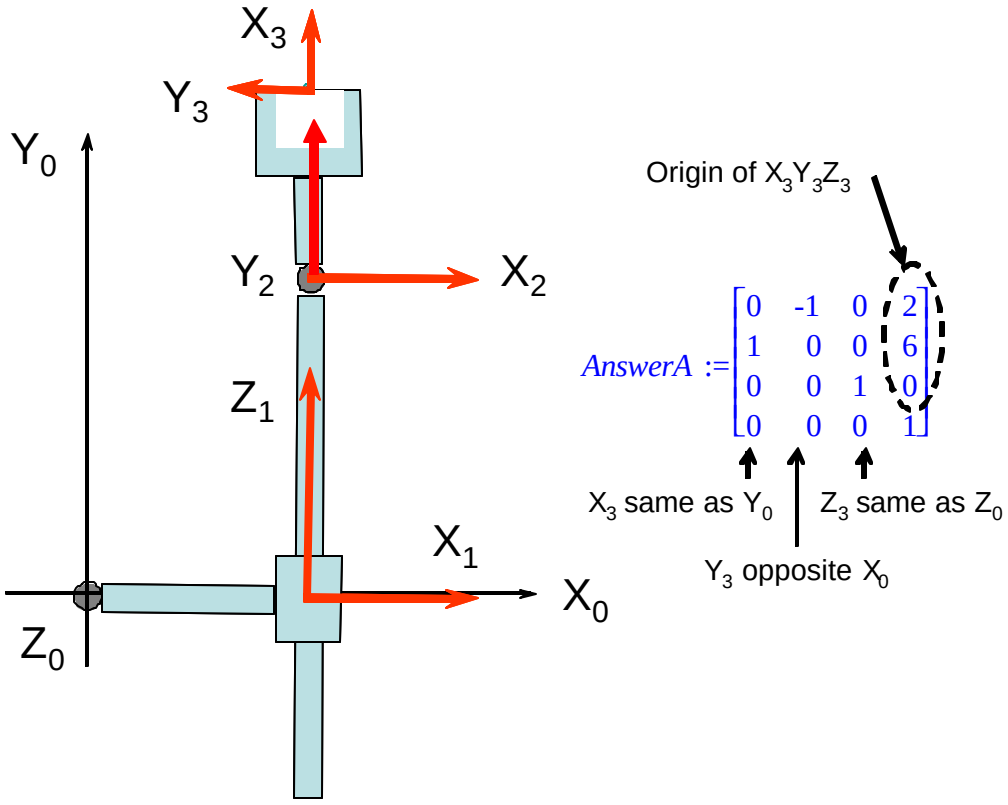
$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos q_3 & -\sin q_3 & 0 & a_3\cos q_3 \\ \sin q_3 & \cos q_3 & 0 & a_3\sin q_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{bmatrix} = \underline{A}_3 \begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{bmatrix}$$

3. Write the overall transformation matrix which relates the final coordinates ($x_3y_3z_3$) to the "base" coordinates ($x_0y_0z_0$) for the RPR Planar Robot (shown below).

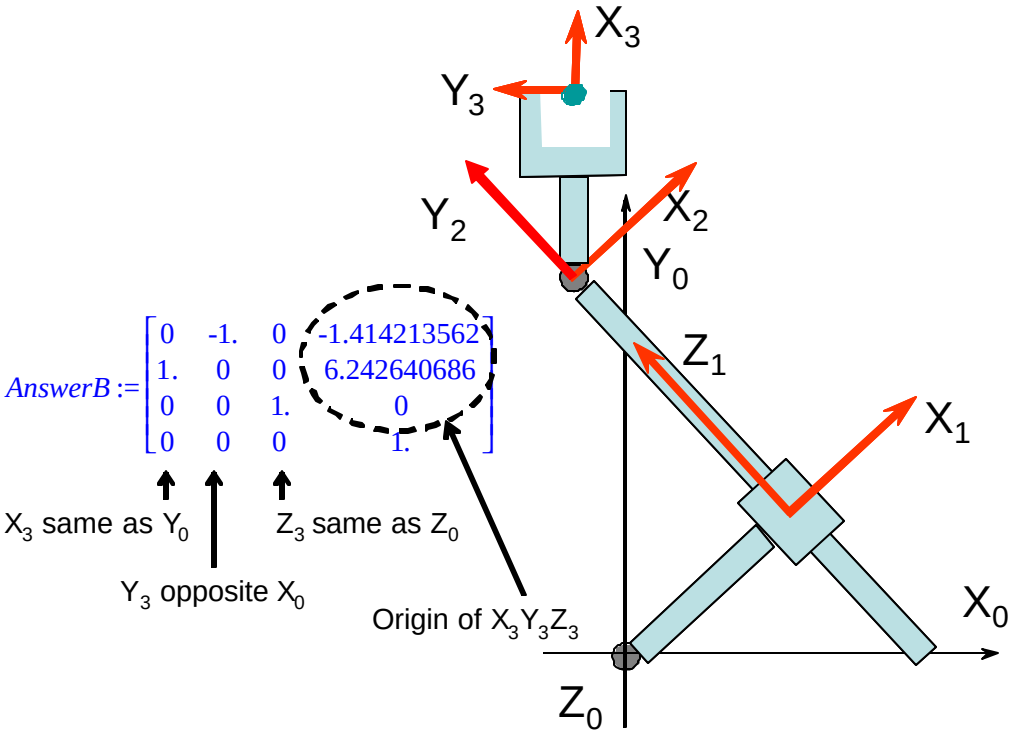
$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos q_{13} & -\sin q_{13} & 0 & +a_1\cos q_1 - d_2\sin q_1 + a_3\cos q_{13} \\ \sin q_{13} & \cos q_{13} & 0 & +a_1\sin q_1 - d_2\cos q_1 + a_3\sin q_{13} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{bmatrix} = \underline{A}_1\underline{A}_2\underline{A}_3 \begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{bmatrix} = {}^0\underline{T}_3 \begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{bmatrix}$$

4. Check the RPR robot in the following configurations

a) $\theta_1 = 0°$, $a_1 = 2$, $d_2 = 4$, $\theta_3=90°$, $a_3 = 2$



b) $\theta_1 = 45°$, $a_1 = 2$, $d_2 = 4$, $\theta_3=45°$, $a_3 = 2$



c) $\theta_1 = 90°$, $a_1 = 2$, $d_2 = 4$, $\theta_3 = 0°$, $a_3 = 2$